Velocity Profiles & D^2_{min} of a Sheared Athermal System with Pins

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Background

- **Add Pins (Tiny particles on a lattice): Influence on Structure and Dynamics**

  A. L. Graves et al, Phys. Rev. Lett, 2016 & Zhang, A.L., et al. Phys. Rev. E, 2022

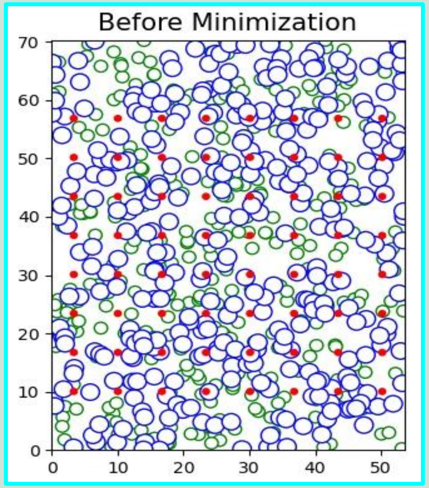
  Wentworth-Nice, P., Ridout, S. A., Jenike, B., Liloia, A., & Graves, A. L. (2020). Structured randomness: jamming of soft discs and pins. Soft Matter, 16(22), 5305-5313.

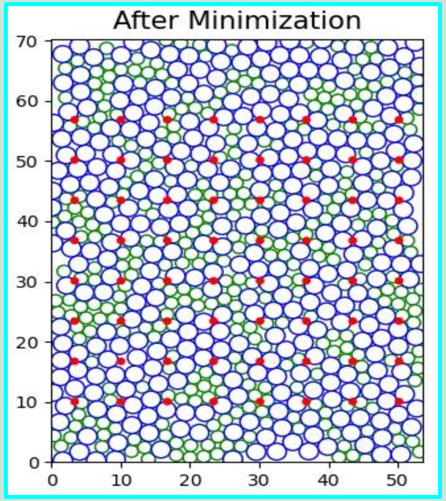
- **Jamming transition at lower packing fraction for higher pin densities**

  A49.00005 : Jamming Transition of Sheared Athermal System With Pins

  N00.00147 : Influence of Pins on The Jamming Transition of a Sheared Athermal System

- **Now:**
  - **Elastic & Transient Macroscopic Properties**
    S01.00007 : Stress Analysis of a Sheared Athermal System with Pins (MIchael J Bolish)
  - My Presentation: **Velocity Profile & Local Rearrangement**





Model

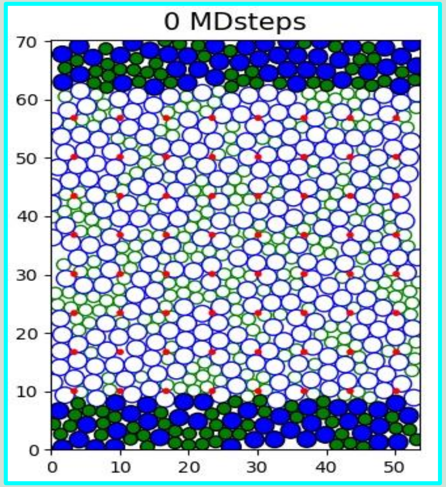
- 2D
- $N_a = N_B = 2048$
- $R_A : R_B : R_{pin} = 1.0 : 1.4 : 0.004$
- $M_A = M_B = M_{pin} = 1.0$
- T = 0

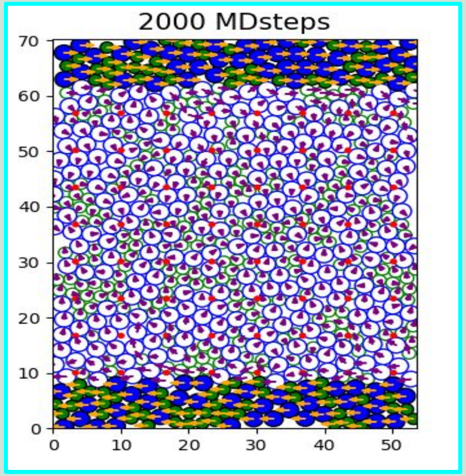
$$r < r_c = R_i + R_j$$

$$\vec{F_{net}} = (F_C + F_D)\hat{r_{ij}}$$

$$F_C = \frac{\epsilon}{r_c^2}(r_c - r)$$

$$F_D = -b(\hat{r_{ij}} \cdot \vec{v_{ij}})$$

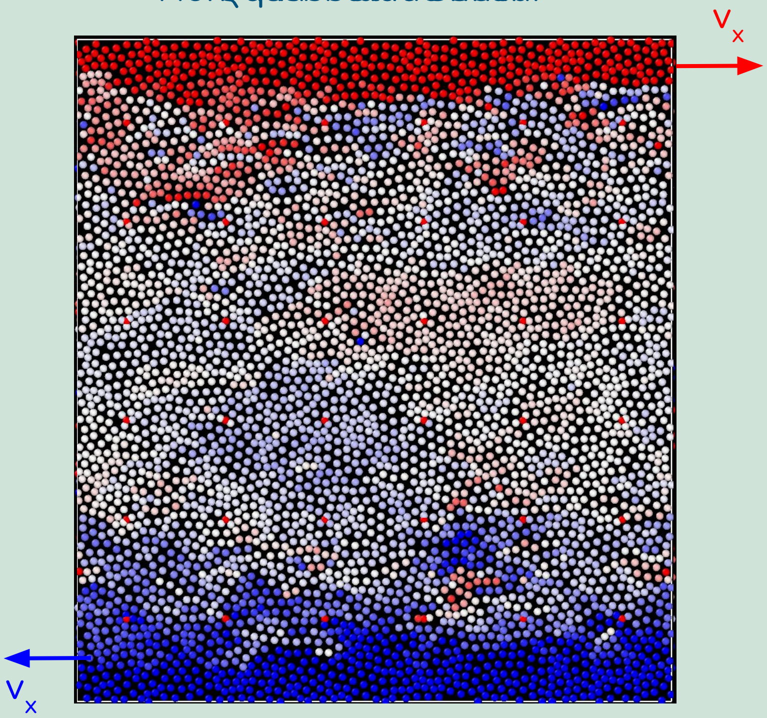


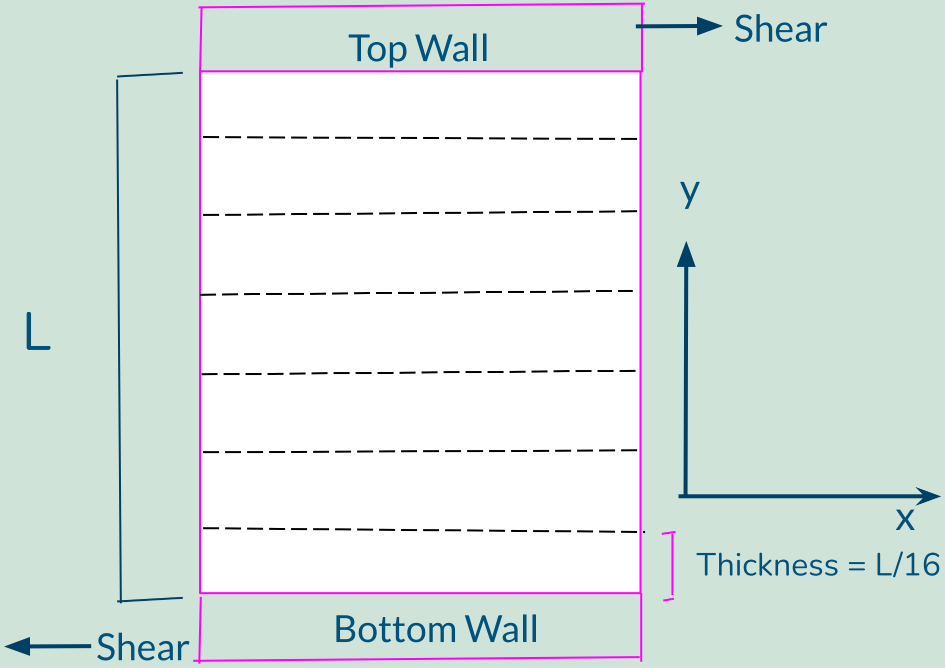


Velocity Profile

Non-quasi-static Shear

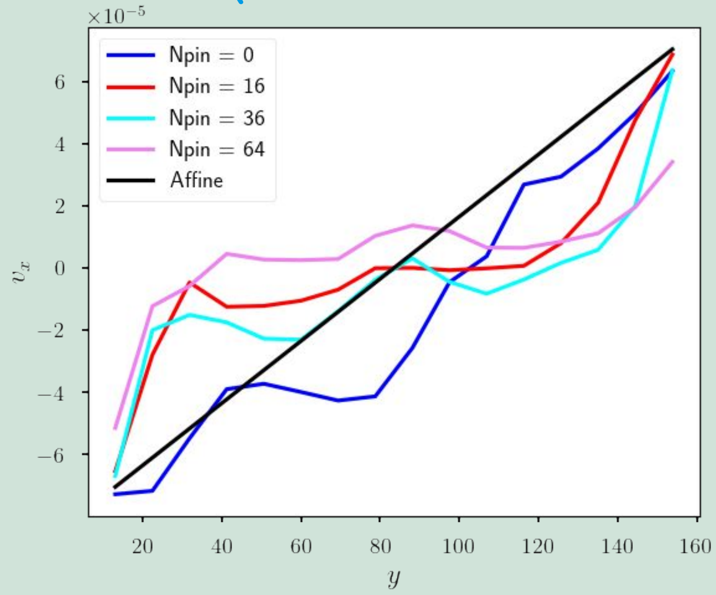
Quasi-static Shear



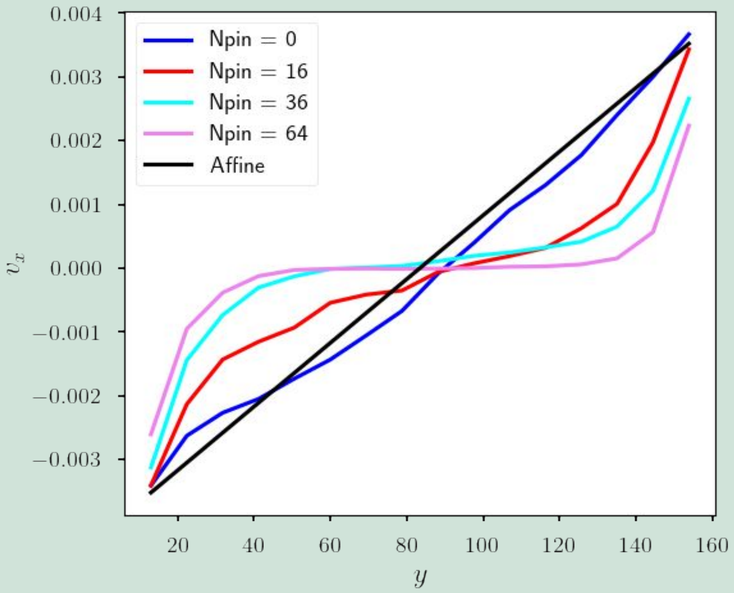


Velocity Profile: Shear Rate & N_{pin}

Quasi-static shear



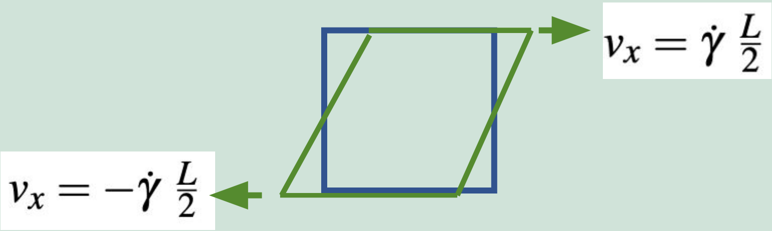
Non-quasi-static shear



- Quasi-static shearing induces more local rearrangement
- Pins impede velocity propagation for non-quasi-static shearing

Harmonic Force Field : Velocity Profile

[B. Tighe, private communication]

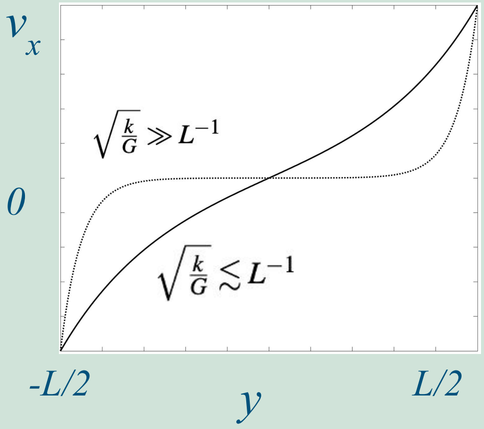


$$\gamma = \frac{\partial u_x}{\partial y}$$

Stress balance: $\frac{\partial \sigma}{\partial y} = G\frac{\partial \gamma}{\partial y} = F_x$

$$G\frac{\partial^2 u_x}{\partial y^2} = \begin{cases} 0 & \text{zero force} \\ k u_x & \text{harmonic force} \end{cases} \quad (1)$$

BC's $u_x = \pm\gamma\,\frac{L}{2}$

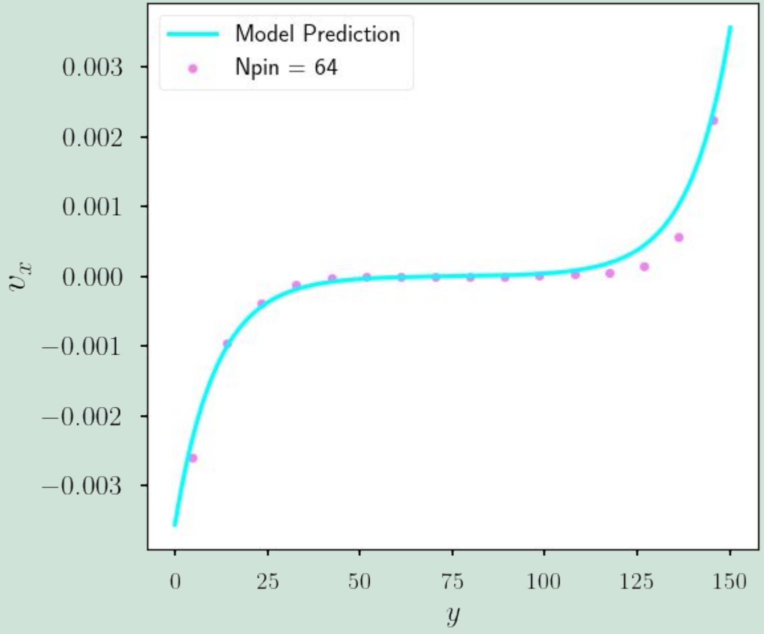


Solution to (1)

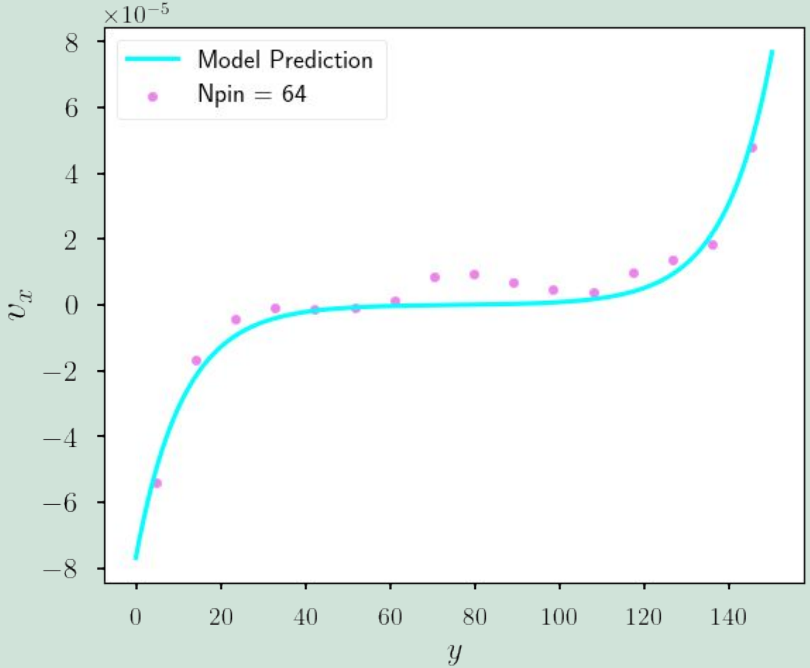
$$\dot{u}_x = v_x = \begin{cases} \dot{\gamma}\, y & \text{zero force} \\ \dot{\gamma}\,\frac{L}{2}\,\frac{sinh(\sqrt{\frac{k}{G}}y)}{sinh(\sqrt{\frac{k}{G}}\frac{L}{2})} & \text{harmonic force} \end{cases}$$

Qualitative Comparison Between Data and Model

Non-quasi-static shear



Quasi-static shear



Velocity profiles qualitatively agree with Harmonic Force Model

To analyze intermittent dynamics microscopically...

$$D^2_{min}$$

D^2_{min} Definition

find the best uniform strain

$$D^2_{min,i}(t,\Delta t) = \min_{\epsilon} \sum_j \left(\vec{r_j}(t) - \vec{r_i}(t) - (\underline{\underline{1}} + \underline{\underline{\epsilon}})[\vec{r_j}(t-\Delta t) - \vec{r_i}(t-\Delta t)] \right)^2$$

actual displacement

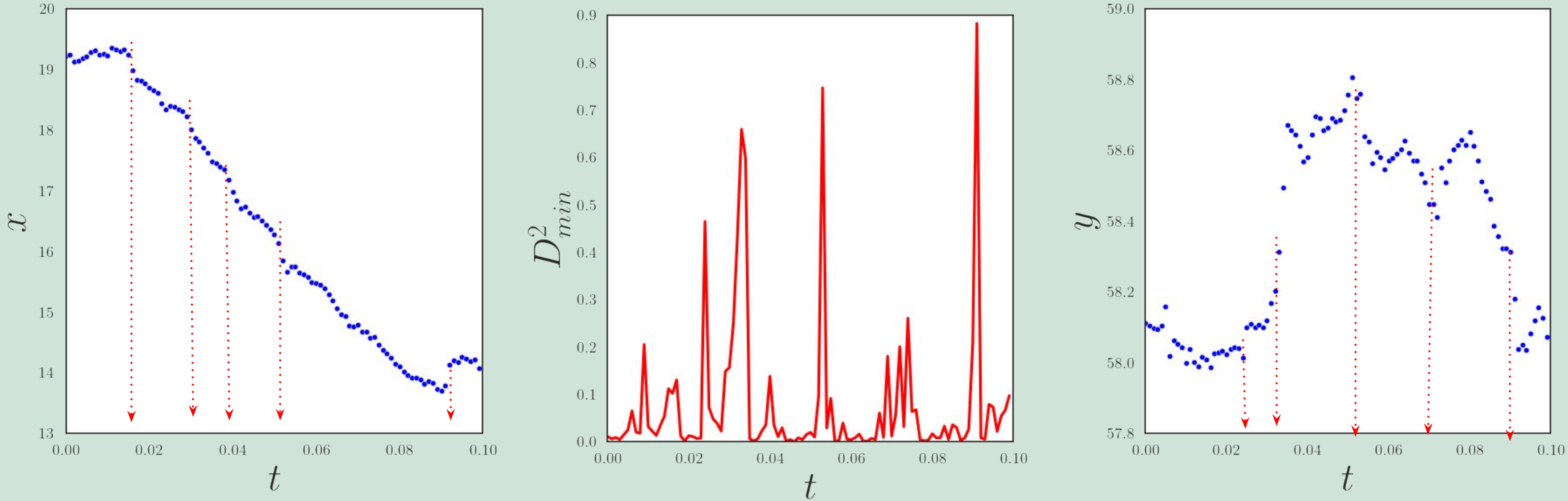
predicted displacement for uniform strain

[Falk, M. L., & Langer, J. S. (1998). Dynamics of viscoplastic deformation in amorphous solids. Physical Review E, 57(6), 7192.]

Technical Details for our system with pins:

- What is the appropriate Δt for calculating D^2_{min}?
- Does Δt vary for different shear rates?

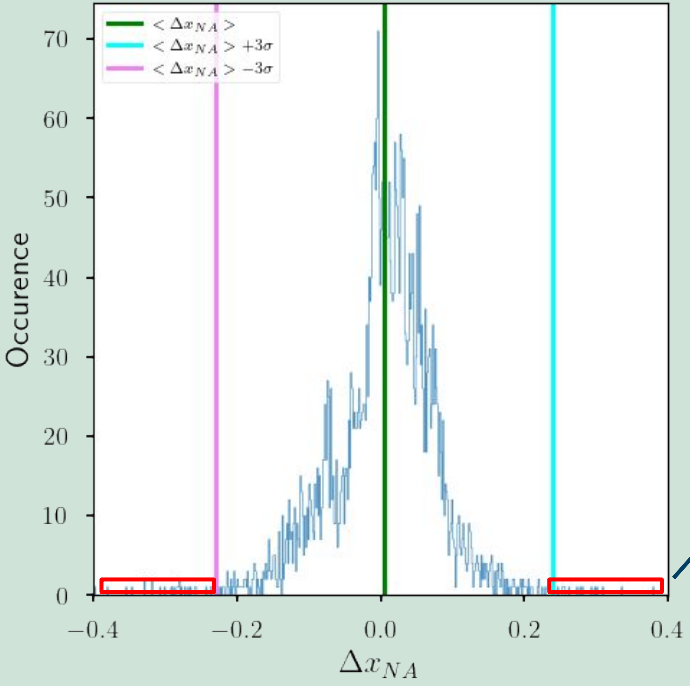
Single Particle Plots

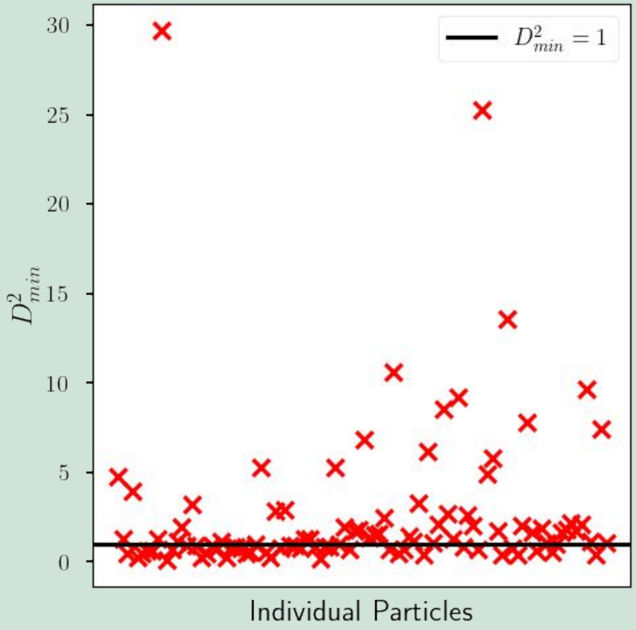


'Jump' in Trajectory → Spike in D^2_{min}

D^2_{min} : Choice of Time Separation

$$\Delta x_{i,NA} = \Delta x_i - v_A(y)\Delta t$$



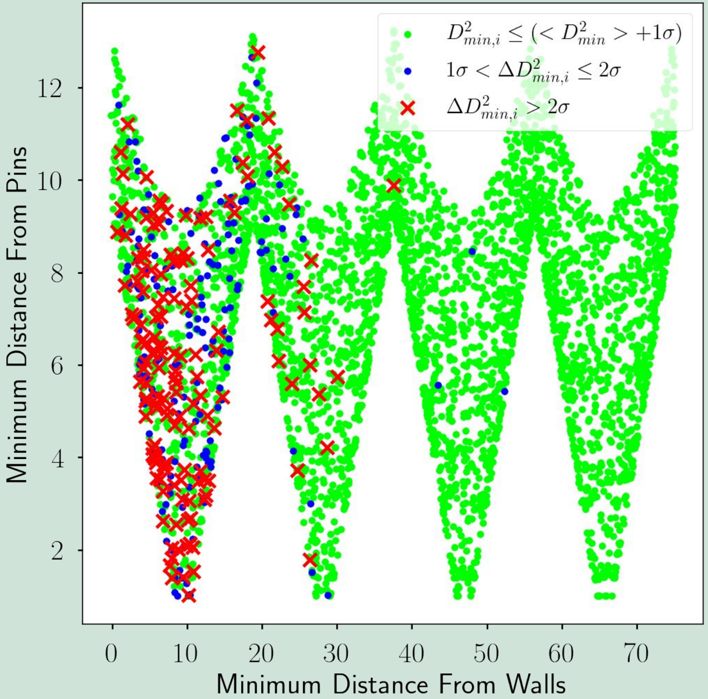


- $\Delta\gamma = \dot{\gamma}\Delta t$
- $\Delta\gamma$~ constant order for different shear rates

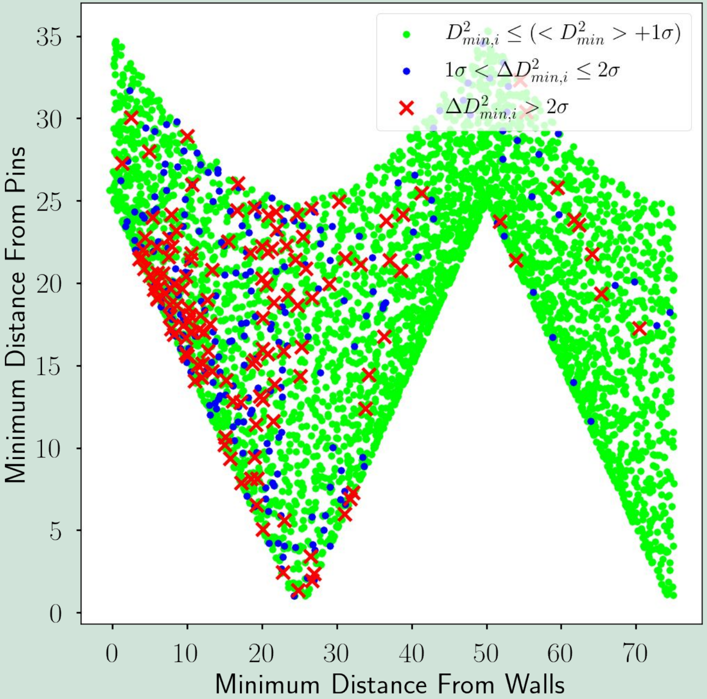
Pins on D^2_{min} : Non-quasi-static Shear

$$\Delta D^2_{min,i} = D^2_{min,i} - < D^2_{min} >$$

N_{pin} = 64

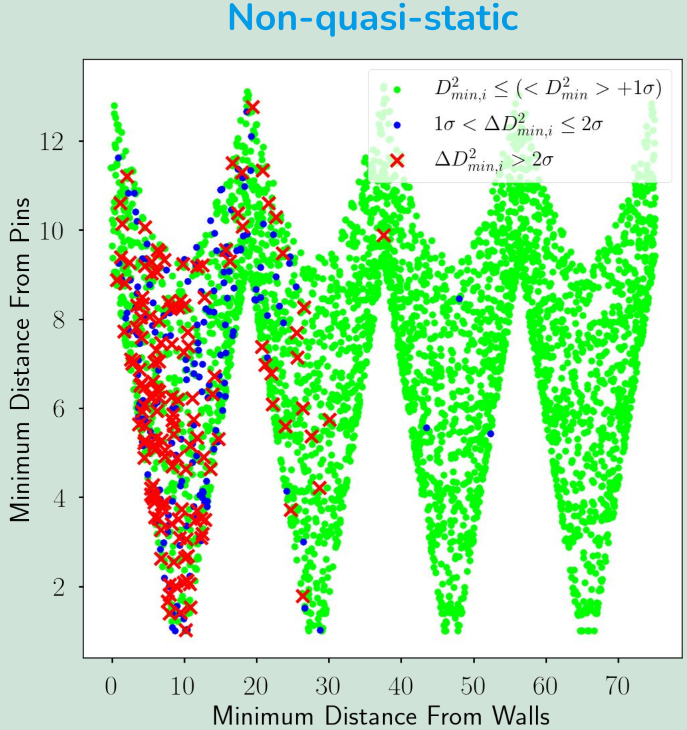


N_{pin} = 9

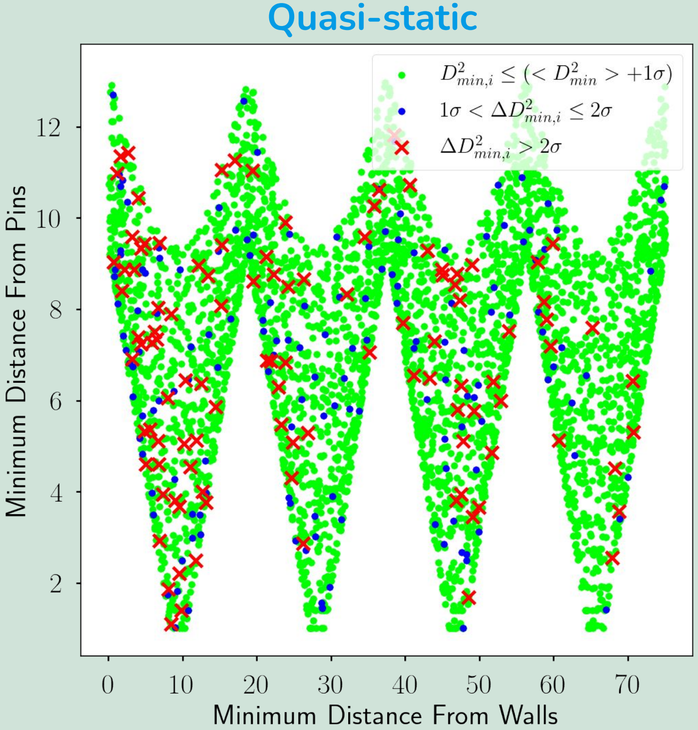


Pins shield the middle layers from rearrangement

Shear on D^2_{min} : Quasi- vs. Non-quasi-static



N_{pin} = 64



Quasi-static shearing allows rearrangement for middle layers

Summary

- ❖ Velocity profiles qualitatively match toy model.

- ❖ For non-quasi-static shearing and higher N_{pin} :
  - More non-affine velocity profiles.
  - Low frequency of higher $D^2{}_{min}$ in the middle layers.
  - Less rearrangement in this region.

- ❖ For quasi-static shearing:
  - (Nonaffine) velocity profiles with more fluctuations in the middle layers.
  - High frequency of higher $D^2{}_{min}$ in the middle layers even for higher N_{pin}.
  - Local rearrangement in this region.

Additional Relevant Presentations

N00.00243 The effect of pins on micro- and macroscopic properties of sheared particles near jamming

Jean Luc Ishimwe & Xiang Li

N00.00160 Shearing of jammed granular systems with fixed pinning sites

Diana Phommavanh

N00.00238 Shear stress and pressure of a granular system with pins

Amin Danesh

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