Approach for E-field Integrals

1. Draw a sketch, choose integration variable (x, y, or θ)

2. Pick a tiny piece of charge and label it dq

   a. label the size of this tiny piece using dx, dy, or $R\,d\theta$

   b. Draw r (distance between dq and P) on the sketch

   c. Draw an arrow for $d\vec{E}$ at P due to dq

3. Find dE magnitude in terms of integration variable:

   a. Find dq. Line: $dq = \lambda\, dx$ or $dq = \lambda\, dy$. Arc: $dq = \lambda\, Rd\theta$

   b. Find r (use Pythagoras)

   c. Plug dq and r into $dE = k\, dq/r^2$

4. Determine the components $dE_x = dE\ \cos\theta$ and $dE_y = dE\ \sin\theta$. You may need to use similar triangles.

5. Determine the limits of integration (where is the charge?)

6. Put it together and solve for $E_x = \displaystyle\int dE_x$ and $E_y = \displaystyle\int dE_y$.

Useful Integrals

$$\int \frac{x\,dx}{(a^2+x^2)^{3/2}} = \frac{-1}{\sqrt{a^2+x^2}}$$

$$\int \frac{dx}{(a^2+x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2+x^2}}$$

$$\int \sin^2(ax)\,dx = \frac{x}{2} - \frac{1}{4a}\sin(2ax)$$

$$\int e^{-bx}\,dx = -\frac{1}{b}e^{-bx}$$

$$\int xe^{-bx}\,dx = -\left(\frac{x}{b} + \frac{1}{b^2}\right)e^{-bx}$$

$$\int x^2e^{-bx}\,dx = -\left(\frac{x^2}{b} + \frac{2x}{b^2} + \frac{2}{b^3}\right)e^{-bx}$$

$$\int x\sin^2(ax)\,dx = \frac{x^2}{4} - \frac{x}{4a}\sin(2ax) - \frac{1}{8a^2}\cos(2ax)$$