Nonconservative Work and Energy

The work-kinetic energy theorem relates the change in kinetic energy of an object to the net work done on the object:

$$W_{\mathrm{net}} = \Delta K. \tag{H.1}$$

This is Eq. (6.14) in Wolfson. We can divide this total work into two categories with W_{cons} the work done by conservative forces and W_{nc} the work done by nonconservative forces. Then the work-kinetic energy theorem becomes

$$W_{\mathrm{cons}} + W_{\mathrm{nc}} = \Delta K. \tag{H.2}$$

As discussed in sections 7.1 and 7.2 of Wolfson, the work done by conservative forces is "stored" in the form of potential energy U, expressed via the relation $W_{\mathrm{cons}} = -\Delta U$. Substituting this in gives

$$-\Delta U + W_{\mathrm{nc}} = \Delta K, \tag{H.3}$$

or

$$\boxed{W_{\mathrm{nc}} = \Delta K + \Delta U = \Delta E_{\mathrm{mech}},} \tag{H.4}$$

where we introduce the mechanical energy

$$E_{\mathrm{mech}} = K + U. \tag{H.5}$$

Eq. (H.4) says that the change in mechanical energy is equal to the work done by nonconservative forces.

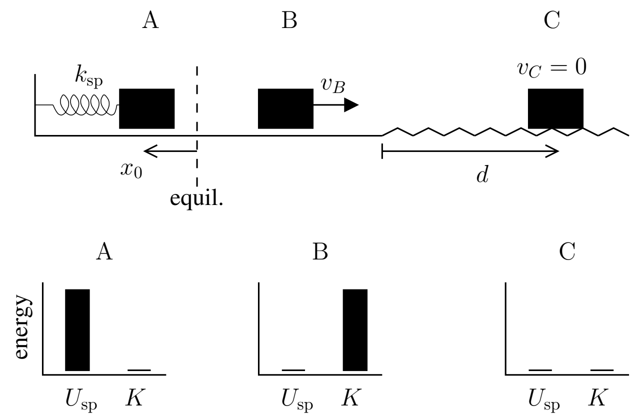
If there are no nonconservative forces or if the nonconservative forces do no work, then $W_{\mathrm{nc}} = 0$ and E_{mech} is a constant. This is the conservation of mechanical energy discussed in section 7.3 of Wolfson. See Eq. (7.5).

When W_{nc} is nonzero, mechanical energy is not conserved. It can be useful to rewrite Eq. (H.4) using $\Delta E_{\mathrm{mech}} = E_{\mathrm{after}} - E_{\mathrm{before}}$ as

$$E_{\mathrm{before}} + W_{\mathrm{nc}} = E_{\mathrm{after}}. \tag{H.6}$$

Example H.1 A Sliding Block

A block of mass m is placed next to a spring with spring constant k_{sp} that has been compressed an amount x_0, as shown. The spring is released, launching the block to slide across a horizontal surface. At first the surface is frictionless, and then the block reaches a rough part of the surface with friction force $\vec{F}_{\mathrm{fr}}$ acting on the block. Eventually it comes to a stop. Solve for the speed v_B on the smooth surface and the distance d traveled on the rough surface in terms of the given quantities.



The mechanical energy consists of spring potential energy and kinetic energy:

$$E_{\mathrm{mech}} = U_{\mathrm{sp}} + K. \tag{H.7}$$

The energy bars show how these energy contributions change. In going from A to B, the total energy is unchanged (since $W_{\mathrm{nc}} = 0$), but has transformed from spring potential to kinetic. In going from B to C, the mechanical energy has decreased to zero, because of the negative work done by friction.

We can set $E_A = E_B$, to get

$$\begin{aligned} \frac{1}{2}k_{\mathrm{sp}}x_A^2 + \frac{1}{2}mv_A^2 &= \frac{1}{2}k_{\mathrm{sp}}x_B^2 + \frac{1}{2}mv_B^2 \\ \frac{1}{2}k_{\mathrm{sp}}x_0^2 + \quad 0 \quad &= \quad 0 \quad + \frac{1}{2}mv_B^2. \end{aligned} \tag{H.8}$$

With a little algebra this gives

$$\boxed{v_B = x_0\sqrt{k_{\mathrm{sp}}/m}.} \tag{H.9}$$

Energy is not conserved once the block reaches the rough surface, because of the nonconserved work

$$W_{\mathrm{nc}} = \vec{F}_{\mathrm{fr}} \cdot \Delta\vec{r} = -F_{\mathrm{fr}}d. \tag{H.10}$$

Now we can set $E_A + W_{\mathrm{nc}} = E_C$ to get

$$\begin{aligned} \frac{1}{2}k_{\mathrm{sp}}x_A^2 + \frac{1}{2}mv_A^2 + W_{\mathrm{nc}} &= \frac{1}{2}k_{\mathrm{sp}}x_C^2 + \frac{1}{2}mv_C^2 \\ \frac{1}{2}k_{\mathrm{sp}}x_0^2 \; + \quad 0 \quad - F_{\mathrm{fr}}d &= \quad 0 \quad + \quad 0, \end{aligned} \tag{H.11}$$

which results in

$$\boxed{d = \frac{k_{\mathrm{sp}}x_0^2}{2F_{\mathrm{fr}}}.} \tag{H.12}$$