# Lab 10

# Measurement of $g$ with a pendulum

### Continuing Objectives

**2.** Know how to determine experimental uncertainties (multiple measurements of the same quantity, propagation of errors, etc.).

**3.** Be able to write an experimental result (including correct number of significant digits, uncertainty, units).

**5.** Know how to keep a clear and organized record, including an introduction (with purpose of lab and appropriate laws or equations), apparatus sketch, table of raw data and calculated quantities, and a good conclusion or summary.

**7.** Know how to make comparisons: are two measured quantities equal? Is a measured quantity statistically equivalent to a theoretical value?

## Introduction

You will measure the gravitational field strength $g$ in the laboratory with precision consistent with the constraints imposed by the available equipment. Your results must include a statement of the accuracy you achieve in terms of a quantitative estimate of the uncertainty in the value you measure. You are provided with a simple pendulum, a stop watch for measuring the period of oscillation $T$, and a meter stick and calipers for measuring the length $L$ (to the center of mass) of the pendulum.

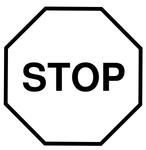
For small amplitude oscillations, the quantities $T$, $L$, and $g$ are related, to good approximation, as follows:

$$T = 2\pi\sqrt{L/g}. \tag{10.1}$$

From this you can determine $g$ as a function of the measured properties of your pendulum, $L$ and $T$:

$$g = g(L,T) = \frac{4\pi^2 L}{T^2}. \tag{10.2}$$

Recall in the experiment *Statistical Uncertainties*, you learned experimental techniques about accurately measuring the period of a pendulum. In your notebook, make a list of all the relevant considerations and methods that improved your measurement.




Discuss your answer with your instructor or TA.

## Reminder About Uncertainties

The total uncertainty in your determination of $g$ is due to two uncorrelated factors: an uncertainty $\Delta T$ in your measurement of period $T$, and an uncertainty $\Delta L$ in your measurement of the length $L$ of the pendulum. Each of these uncertainties contributes to the total uncertainty $\Delta g$ in the calculated value of $g$. To figure out the total uncertainty $\Delta g$, you will first calculate the quantity $\Delta g_L$, the uncertainty due to uncertainty in $L$ alone, and the quantity $\Delta g_T$, the uncertainty due to uncertainty in $T$ alone. You can calculate the uncertainties $\Delta g_L$ and $\Delta g_T$ by explicitly calculating the effect on the value of $g$ by using the extreme value of the variables $L$ and $T$ in Eq. (10.2), e.g.,

$$\Delta g_T = g(L, T + \Delta T) - g(L,T) \tag{10.3}$$

and

$$\Delta g_L = g(L + \Delta L, T) - g(L,T). \tag{10.4}$$

(For the propagation of uncertainties see Appendix A.1 and also Lab 2) You can combine these uncertainties from the independent measurements of $L$ and $T$ using Eq. (A.4) of Appendix A.1.

## Procedure

1. Measure the length $L$ of your pendulum. How precisely do you know this length? Considering this precision, assign a reasonable value for the uncertainty $\Delta L$ in your measurement of the length.

2. You have many choices in your method of measuring the period. To explore these options, begin by completing the following task. Carefully measure the time for a single complete oscillation of your pendulum. Repeat this measurement of the period of your pendulum 11 times so that you have a total of 12 values for the period of your pendulum. Calculate the mean $\langle x \rangle$ and the experimental standard deviation $s$ of your 12 measurements.

   **Question:** What information does your particular value of $s$, the experimental standard deviation, tell you about your timing data?

3. Report your best value of the period of your pendulum in the form:
   $T =$ ______ $\pm$ ______.

   **STOP** Discuss your answer with your instructor or TA.

   **Question:** The value of the period you just measured is based on 12 swings. Suppose you decide to measure not 12, but 120 swings. Without doing any analysis at this point, which of the following methods do you think would give the smallest uncertainty?

   (a) Method A: Ten separate measurements of the time for 12 swings.

   (b) Method B: One measurement of the time for 120 swings.

   (c) It doesn't matter, both methods give the same uncertainty.

   **Task:** Write your answer in your lab notebook so you can return to it later.

Now let's do the analysis to determine how best to improve the uncertainty in our measurement of the period of the pendulum. In the following we will use method A in steps 4 — 7 and method B in steps 8 — 9.

**Method A:**

4. Carefully measure the time for 12 complete oscillations of the pendulum. Repeat this measurement 9 more times so that you have a total of 10 values for the time taken for 12 periods. Calculate the mean and the experimental standard deviation, $s$, of your 10 measurements.

5. Calculate your best estimate for the time taken for 12 complete oscillations. Be sure to include your experimental uncertainty for this value.

6. Using your results from step 5, state your best estimate for the period of your

pendulum $T$. Be sure to include your experimental uncertainty for this value.

**STOP** Show the results of your calculations in steps 5 and 6 to your instructor or TA.

7. Calculate a value for $g$, $\Delta g_L$, $\Delta g_T$, and $\Delta g$. Give your final answer for the gravitational field strength in the form $g \pm \Delta g$. Which measurement contributes the most to your uncertainty?

**STOP** Show the results of your the calculation of $g \pm \Delta g$ to your instructor or TA and discuss your conclusions on which measurement contributes most to the uncerainty.

The uncertainty in length of the pendulum is fixed by the measuring apparatus at hand, but you can reduce the uncertainty $\Delta T$, and thus $\Delta g_T$ and $\Delta g$, by timing over more periods. This is because the same timing error introduced when the stopwatch is started and stopped will be spread out over more periods.

**Method B:**

8. Carefully measure the time it takes to complete 120 full swings of your pendulum, and determine new values for $T$ and $\Delta T$. Recall that you have a value for $s$ from step 2. This last measurement is also a single timing measurement; think about how your $s$ from one swing relates to your uncertainty in the time for 120 swings. Now, determine $\Delta T$, the uncertainty in time for one of the 120 swings.

   **Question:** Based on your results for the methods of measuring $T$ in step 6 and step 8, which method gave the smallest uncertainty $\Delta T$? Does this agree with your initial guess in step 3?

9. Calculate a new value for $g$, $\Delta g_L$, $\Delta g_T$, and $\Delta g$. Give your final answer for the gravitational field strength in the form $g \pm \Delta g$.

   **Question:** Which measurement contributes the most to your uncertainty?

**STOP** Show your final results for the measured value of $g$ to your instructor or TA. Be prepared to explain the difference between this result for $g$ and that found in step 7.

10. If you measured the time it takes to complete 1200 swings (*don't actually take the time to do this!*), estimate what values you would get for $\Delta T$, $\Delta g_T$, and $\Delta g$. Is it worth timing 1200 swings?

Prepare a conclusion reporting your best measurement for $g$ in correct scientific notation. Also include a reflection on what you have learned about experimental procedures that improve the precision of experimental results.