Lab 6

Numerical Simulation: Air Drag on a Falling Object

Continuing Objectives

6. Be able to make a good graph either in your notebook or with a computer (label, scales, units, dependent, and independent variables).

9. Use a computer to simulate physical systems using numerical methods.

Lab-specific goals

1. Learn to numerically simulate models for a physical phenomenon using Euler's method and Newton's second law.

2. Learn to compare data from such simulations with experimental data to find out how well a particular model works, and find unknown model parameters.

In this lab...

...we will simulate two models for air drag and determine which model better reproduces a particular set of data taken from an experiment. In the experiment, an object is dropped from a set height, and recorded. Video tracking software is used to obtain the object's position with respect to time, from which velocity and acceleration can be computed.

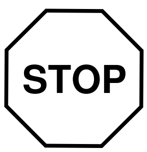
As always, feel free to call over a TA or instructor at any point during this lab, not just at the stop signs!

Think and reflect

Spend a few minutes thinking about the following questions:

1. How have you experienced air drag or air resistance in your daily life?
2. If we think about air drag as a force, what might the **magnitude** and **direction** of this *drag force* depend on? It might be helpful to use your own experiences as a starting point.

Now, using your answers to the above questions as a starting point come up with a set of things that drag force depends on. **Write these down in your lab notebook.**

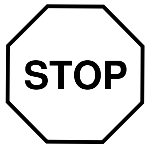


Discuss what you have determined drag force may depend on with a TA or instructor before you move on.

Concepts

Here are some useful concepts about drag:

1. It is caused by relative motion between an object and the surrounding medium, i.e., air, water, etc. We'll focus on air drag.
2. As the object moves, it experiences a force as it bumps into air molecules (i.e., nitrogen, oxygen, water, etc.). This force depends on:
   (a) the **velocity** of the object: a faster moving object experiences a greater drag force.
   (b) the **density** of the air: denser air means more molecules bumping into the object, and therefore, greater drag force.
   (c) the **shape** of the object: Is it aerodynamic? Is it chunky? Is it a plane? A bird? The effect of the shape is usually captured by a number called the *drag coefficient*. A larger drag coefficient leads to a greater drag force. Aerodynamic objects like airplanes have small drag coefficients, for example.
   (d) the **cross-sectional area** of the object: If an object presents a larger cross-section to the air, more air molecules bump into it, increasing drag force.



How do the above concepts compare with what you came up with for the factors drag force may depend on? Take a few minutes to reflect on any significant differences between your predictions and the above list. Feel free to ask a TA or instructor any remaining questions.

Models for drag

Air drag is a complicated phenomenon. In order to study its effect, we have to make a simplified model for it, as we often do in physics. There are two commonly used models for drag force, we'll call them Model A and Model B. As you might've figured out, **drag force always opposes the motion of the object** (assuming the air itself is still).

In Model A, the magnitude of drag force is proportional to the speed (velocity):

$$|\vec{F}_A| = b_A|\vec{v}|, \tag{6.1}$$

where b_A is a proportionality constant that depends on the density of the medium (in our case air), and the shape and cross-sectional area of the object. **What are the units of b_A?**

In Model B, the magnitude of drag force is proportional to the square of the speed (velocity):

$$|\vec{F}_B| = b_B|\vec{v}|^2, \tag{6.2}$$

where b_B is a constant similar to b_A but with different units. **What are its units?**

Even though these are simplified models, the resulting 'equation of motion', i.e., the equation you obtain from applying Newton's second law, is a little complicated to solve directly, so we'll study the problem numerically.

*Note: We refer to b_A and b_B as **drag parameters** (or drag coefficients). Remember that b_A and b_B carry different units.*

Newton's second law – equation of motion

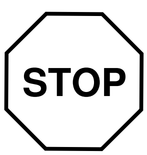
1. Draw a free body diagram for an object falling a distance of y_0 under the influence of gravity and drag force.
2. Using Newton's second law, obtain an equation for the acceleration at any point during the fall. For now, use F_d as the **magnitude** of the drag force in general, *noting that it is **not** a constant force like friction!*. Later, we'll select a specific model to explore more deeply.

Symbol	**Description**	**Test value**	**Units**
g	gravitational field constant	10	$\mathrm{m/s^2}$
m	mass of object	3	kg
b_A	linear drag force parameter	2	N·s/m
b_B	quadratic drag force parameter	2	kg/m
y_0	initial position	0	m
$v_{y,0}$	initial velocity	0	m/s
t_0	starting time	0	s
Δt	time step	0.1	s

Table 6.1: Parameter table for air drag simulation.

Now, do the following:

1. Sketch a plot of position y, velocity v_y, and acceleration a_y vs. time t for the case when there is **no** drag.
2. Now, on the same set of plots, use your intuition to sketch how the graphs would change in the presence of air drag. Make annotations on your sketches. How might you check to see that your intuition is not obviously wrong?[1] (*e.g., What can we expect for the time the object takes to reach the ground? How might this influence the position or velocity curves?*)
3. Include both of these sketches in your lab notebook document.



Discuss your equation, sketches, and responses with a TA or the instructor before you move on.

Numerical solution

To simulate the models described above, we will use a numerical iteration method described in your PHYS 211 Supplemental Reading booklet. In short, you will generate a table (see Table 6.2) containing values for t, y, v_y, and a_y for the falling object. Each row represents a *time step* in the simulation, starting with $t = t_0$ (usually taken to be 0). Once you provide the initial conditions, the computer can fill out the rest of the table using the appropriate stepping equations and expression for acceleration.

To ensure that we've provided the right instructions to the computer, and that

[1] It is always useful to do some reality checks to make sure we're not totally off!

t	$y(t)$	$v_y(t)$	$a_y(t)$
t_0	y_0	$v_{y,0}$	

Table 6.2: Trial calculation using numerical iteration methods.

the 'code' is working correctly, we will do a few steps of the calculation by hand. First, we need to specify values for several parameters. A test set is provided in Table 6.1.

Now, we fill out a table similar to what we did for the section on numerical iteration of the PHYS 211 Supplemental Reading.

1. Make two tables like Table 6.2 in your lab notebook, one for Model A and another for Model B. Be sure to label them with the model they represent.

We are going to fill in a few steps of the simulation by hand using the *Test Parameters* from Table 6.1.

2. In the first row of both tables, fill in the starting values for the corresponding variables from Table 6.1. For a_y, use the values of t, y, and v_y from the **same** row in the equation of motion you derived above (replace the force F_d with that from model A for the Model A table and that from model B for the Model B table).

3. For the second line, use the following stepping equations to calculate the new values for t, y, and v_y:

$$t_\mathrm{new} = t_\mathrm{old} + \Delta t, \tag{6.3}$$

$$y_\mathrm{new} = y_\mathrm{old} + v_\mathrm{y,old}\,\Delta t, \tag{6.4}$$

$$\text{and} \quad v_\mathrm{y,new} = v_\mathrm{y,old} + a_\mathrm{y,old}\,\Delta t. \tag{6.5}$$

   where "old" and "new" refer to values in the previous and the current row, respectively. Enter the values in the corresponding boxes.

4. In the box for a_y, as before, use the values of t, y, v_y from the **same** row in the equation of motion.

5. Repeat this process for the remaining rows to get **four** rows in all for each table.

STOP When you have filled out the tables, show your results to your instructor or TA.

Using Excel to carry out the iterative process

We will now create an Excel spreadsheet to automate the above process, so we can quickly calculate many time steps.

1. Open the Excel spreadsheet template (in `PHYS211_212 Lab/211Lab/Lab 06 - Numerical Simulation/`). Fill in the *Parameters* section with the values from Table 6.1.

   NOTE: Be sure to populate your table only by referencing cell values from the Parameters table. There should be NO PLACE in your spreadsheet table of [t, y, v, a] where a numerical value is entered.

2. Test your spreadsheet by entering the values and formulas you used in the first two rows of Table 6.2 into the first two rows of the *Simulation Data (Model A)* and *Simulation Data (Model B)* sections of the Excel spreadsheet. Ensure that you use appropriate cell referencing and formulas for the iterations. For constants, remember to use **absolute cell referencing** when referencing those values from the *Parameters* table of your Excel spreadsheet. Check with your TA or instructor if you have questions.

3. Click and drag from your second row to fill two more rows in each section. Check that your Excel calculations match your hand-calculated tables. If they do, then you have probably set up your Excel sheet correctly. If they don't, then there is a mistake in your Excel sheet that must be corrected. If you are having difficulty, consult your instructor or TA.

STOP Show your results to your instructor or TA before continuing.

Simulation and Analysis

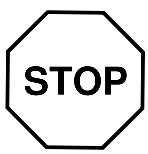
Now that your Excel sheet is correctly set up, you can use it to simulate real data from a falling object. We will generate data for both models and play around with the drag parameters to see if we can fit a particular model to the data given.

1. First, in the *Parameters* table, replace the test values for m and g with their experimental values of $m = 3.8$ g and $g = 9.81$ m/s^2. Also decrease the value of time step Δt to 0.001 s. **Question.** Why is this an important step?

2. The Excel template should contain experimental data for the object that fell in the given video (available to watch in the same directory). These are the data to which we would like to compare our two models in order to establish which model better fits the experimental data.

3. In the *Parameters* table, replace the test values for t_0, y_0, and $v_{y,0}$ with values of the experiment using the line for $t_0 = 0.033$.

4. Now, go to your simulation data and click and drag to fill the formula for t down to enough rows so that you have a total duration that matches the experimental data. Then do the same for y, v_y, and a_y. Do this for both models.



Check in with your instructor or TA if you're having any trouble getting all this to work; otherwise proceed. Now is also a good time to stop and make sure you understand everything we've done so far.

We will now compare the experimental data and the simulation from both models graphically and experiment with the drag coefficients to see if we can fit a particular model to the data given.

5. Make plots in Excel of your simulation results for y vs. t and for v_y vs. t (from the previous step). Make a separate plot for each model and label them Model A or Model B.

6. On your y vs. t and v_y vs. t plots, include the experimental data given. You can plot two data sets on the same plot in Excel by right-clicking any plot and clicking "Select Data". From here, you can add another set of $x-$ and $y-$values to the plot. Experimental values are typically plotted as scatter plots without lines, so follow this convention by plotting the experimental data as unconnected data points. Since you should have set t_0, y_0, and $v_{y,0}$ values identical to the experiment, your simulation line and experimental data points should cross at this point. If they do not, check your initial values match across all three data sets.

7. Now, working first with model A, and then model B, try to adjust the drag parameters to find the value that brings you the closest match between simulation and experimental data. Try to make the model and experimental data

points line up as well as you can on the plots only by changing the drag parameter in each model.

8. Which model works better? That is, which model fits the experimental data better?

Why two models?

The two models presented here work in different regimes of motion, one in which the flow of air around the object is smooth, called the *laminar* regime, and another in which the flow is choppy and turbulent, called the *turbulent* regime. What regime more accurately describes a given situation depends on many factors such as velocity, density, viscosity, shape, and size. It is also possible to switch regimes as the object picks up speed. Do you think you might be better able to model this experiment by using model A for some part and model B for the rest?

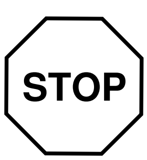
Results and Conclusion

1. Write down the quantitative results for the drag parameters from your analysis. Although there is no uncertainty in this analysis, comment on the goodness of the fit for each model. For your best fitting drag parameters, which model fit the data best overall?
2. Write down a conclusion based on your analysis and results.

Reflection

Respond to the following in a few sentences each:

1. Explain to a friend who's not taking this class the main ideas and concepts in this lab as you understand it.
2. What is the most significant thing that **you** took away from this lab?
3. What was the most challenging aspect of this lab? Spend a few minutes thinking about that aspect, formulate a question that might help you address it, and ask a TA or the instructor your question.



Discuss the above with a TA or instructor before you leave!