Lab 3

Statistical Uncertainties

Continuing Objectives

2. Know how to determine experimental uncertainties (multiple measurements of the same quantity, propagation of errors, etc.).

3. Be able to write an experimental result (including correct number of significant digits, uncertainty, units).

5. Know how to keep a clear and organized record, including an introduction (with purpose of lab and appropriate laws or equations), apparatus sketch, table of raw data and calculated quantities, and a good conclusion or summary.

7. Know how to make comparisons: are two measured quantities equal? Is a measured quantity statistically equivalent to a theoretical value?

Specific Objectives

1. Know the meaning of average, standard deviation, standard deviation of the mean, and distribution.

Introduction

Successive measurements of any physical quantity always have some variation if read to enough significant figures. This variation can be caused by a number of factors, such as vibrations of mechanical parts, air currents, manufacturing variation between apparently identical components, variations in room temperature or humidity, electronic noise in instruments, and even random motions of atoms and molecules. An experimental result therefore always includes an estimate of these variations. In this lab, you will observe variations in repeated measurements. You will also learn how to report an experimental result accurately.

Student Number	Number of blue M&Ms
1	
2	
3	
.	

Table 3.1: Table to record class results for number of blue M&Ms in a bag.

Part I: Statistics of M&Ms

Procedure (Part I)

How many blue M&Ms are in a standard package? While this might seem like a mundane thing to investigate, answering this question requires the understanding of statistical analysis techniques that have a wide variety of applications, such as interpreting quality control data in industry or monitoring changes in a hospital patient's lab results.

In this lab, each lab member will count the number of blue M&Ms in their bag to obtain a set of class-wide data. We will then gain some experience describing this data set using statistical analysis. You will use the principles learned in today's lab in other labs (and likely in other courses), so be sure to take good notes and ask questions if anything is unclear.

1. Each student in the lab will be given a small bag of M&Ms. Open the bag over a clean piece of paper, count the number of blue M&Ms in your bag, and record the number in the shared Google Sheet (link on the PHYS 211 website on the Lab Info page) as in Table 3.1.

2. Examine the table of values. Without doing any calculations, what is your estimate for the average number of blue M&Ms in a typical bag? How would you characterize the fluctuations in the values of your table? These fluctuations are a measure of the differences over the sample in the number of blue M&Ms in a single bag.

3. To interpret these data graphically, plot the number of blue M&Ms versus the student number by making a scatter plot without lines connecting data points. Print and paste a copy of this plot into your lab notebook. Does examination of this graph cause you to change your previous estimates made in step 2? Comment in your notebook.

Number of blue M&Ms	Number of bags
0	
1	
2	
3	
.	

Number of blue M&Ms	Number of bags
0	0
.	.
8	3
9	4
.	.

Table 3.2: Data table for histogram. The sample data discussed in the manual is entered in the table on the right.

4. Next we answer the first question posed in step 2 quantitatively. As the best estimate for the number of blue M&Ms in a typical bag, calculate the average value:
$$\langle x \rangle = \frac{1}{N}\sum_{i=1}^{N} x_i \tag{3.1}$$
where x_i is the ith value of x (that is, the 1st data point is x_1, the 2nd is x_2, and so on). Use the Excel function `SUM` to determine the sum.

   Check that using the Excel function `=AVERAGE(cell 1:cell N)` will give you the same value of $\langle x \rangle$ that you previously calculated. In future labs, you may use this Excel function `AVERAGE`. Next draw your result for $\langle x \rangle$ as a horizontal line on your plot from step 3. The average $\langle x \rangle$ is also called the *experimental mean.*

5. Your plot from step 3 is one way to look at the complete set of values. Note that your plot includes more information than just the experimental mean; the plot also includes the fluctuations. There is another way of graphically representing your data called a *histogram.* Make a second table in your Excel spreadsheet as shown in Table 3.2.

   The first column is the number of blue M&Ms in a bag. The second column tells you how many bags had that specific number of blue M&Ms. For example, if no student found a bag with 0 blue M&Ms, 3 students found bags with 8 blue M&Ms, and 4 students found bags with 9 blue M&Ms, then your table would have the entries shown on the right in Table 3.2. You can use the Excel function `=COUNTIF(cell 1:cell N, condition)` to do this. For instance, to determine the number of students finding 4 blue M&Ms, enter `=COUNTIF(A1:A30, 4)`. You may even replace the `4` with a cell reference, so that you can copy the formula for the whole column.

Complete the entries in your Excel version of Table 3.2 which corresponds to the M&M data obtained in step 1. Check that your numbers in the second column add up to the total number of students.

6. Now plot your data from your version of Table 3.2 (number of bags versus number of blue M&Ms). Note that to make a histogram in Excel, first plot the data as a "Scatter" plot, then change the "chart type" to "Column". (*Note: using the built-in histogram plot will automatically bin (or group) the data, which is something we do NOT want in this case.*) This plot is the *histogram* of your data.

7. Print and paste a copy of this histogram plot into your lab notebook. Draw in a vertical line at the $\langle x \rangle$ value you obtained previously.

   In step 3 we asked you to characterize the fluctuation in the values of Table 1. How would you graphically represent these fluctuations on your histogram?

8. Now we are ready to address these fluctuations quantitatively. We want a quantity which tells us how much the x_i values differ from $\langle x \rangle$, so let's consider the quantity, $(x_i - \langle x \rangle)$. However, we do not care about the sign, so let's take the square of this quantity, $(x_i - \langle x \rangle)^2$. It would then make sense to take the average of these values by summing all N values of the quantity above and dividing by N, where N is the total number of independent pieces of information that are used in the calculation. However, as we saw in Eq. 3.1, the calculation of $\langle x \rangle$ depends on x_i. Because of this, there are now $N-1$ independent measurements in the calculation; for example, x_1 could be determined from $\langle x \rangle$ and x_2, x_3, x_4, and so on. Therefore, we instead divide by $N-1$ (the term $N-1$ is called Bessel's correction). Finally, we must take the square root of this expression. We call this resulting quantity the experimental standard deviation:

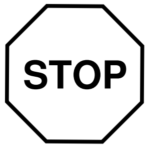
$$\begin{array}{c}\text{experimental}\\\text{standard deviation}\end{array} \quad s = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \langle x \rangle)^2}. \tag{3.2}$$

9. Add to your Table 3.1 a third column that calculates values of $(x_i - \langle x \rangle)^2$. Using the values in this third column, determine the experimental standard deviation, s, using Eq. (3.2) and the Excel function `SQRT`.

10. Check that using the Excel function `=STDEV(cell1:cellN)` gives you the same value of s that you previously calculated from your spreadsheet. If it does, then that means that you may use the Excel function `STDEV` in future labs.

    Compare this new value of s with your previous estimates of fluctuations in steps 3 and 7. Remember that these fluctuations are a measure of the uncertainty in the number of blue M&Ms in a single bag.

11. You are now ready to answer the question: "If you were to open one single new M&M bag, what would be your prediction for the number of blue M&Ms in this bag?" Report your result in the format you learned at the end of Lab 2 (also described in Appendix A.2.)

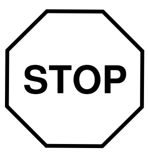


Show your result to your instructor or TA before continuing.

12. You can transform your histogram into a probability distribution of blue M&Ms. To do so, add to your Table 3.2 another column which is

$$P(x) = \frac{\text{number of bags with } x \text{ blue M\&Ms}}{\text{total number of bags}}. \tag{3.3}$$

Plot a graph of $P(x)$ versus the number of blue M&Ms, x. Describe in words the meaning of $P(x)$.



Show your conclusions to your instructor or TA before continuing.

Theory

The universe of possible measurements

So far we have only opened one M&M bag for each student (N bags of M&Ms in total). In order to fully characterize the complete distribution of blue M&Ms in this batch, our goal would be to open every single M&M bag produced in this batch. Besides getting a stomach pain from eating all these M&Ms, this task is obviously impossible. Even so, we won't give up. Instead, we imagine how the probability distribution $P(x)$ of the blue M&Ms of all (let's say $N \to \infty$) M&M bags would look.

We call this distribution *the distribution of the universe of possible measurements.* Just as before it would have a mean value and a standard deviation, the true mean value μ is:

$$\mu = \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} x_i, \tag{3.4}$$

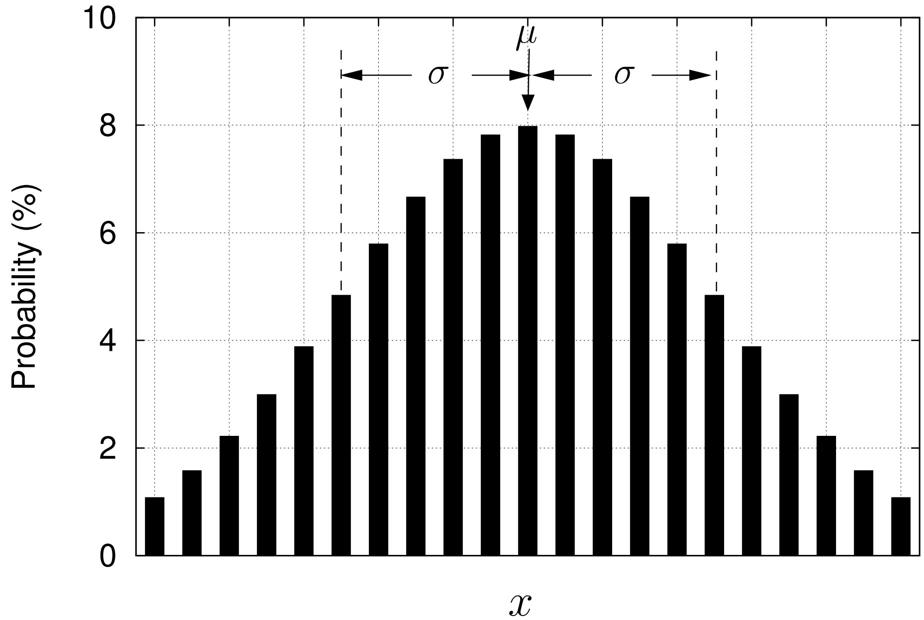


Figure 3.1: Probability distribution of the *universe of possible measurements*.

and the true standard deviation σ is:

$$\sigma = \lim_{N\to\infty} \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i-\mu)^2}, \tag{3.5}$$

as illustrated in Figure 3.1.

When the distribution has the typical Gaussian shape of Figure 3.1, approximately 68% of the measurements lie within $\pm\sigma$ of the true mean, 95% within $\pm 2\sigma$ of the true mean, and 99.74% within $\pm 3\sigma$ of the true mean.

Your job as an experimenter is to estimate the true mean and the true standard deviation from just a few measurements. If you make just one measurement, that measurement has a 68% chance of lying within $\pm\sigma$ of the true mean. But this statement tells you nothing because neither the true mean nor the true standard deviation is known ahead of time. In fact, a single measurement, while giving a ball park estimate of the true mean, says nothing at all about the true standard deviation. It is for this reason that, almost always, you must make *multiple* measurements.

Estimates of the true mean and true standard deviation

The best estimates of μ and σ that can be obtained from a set of N measurements are given by the experimental mean and standard deviation of your N measurements. The experimental mean and standard deviation are exactly what you have determined so far in this lab, i.e., as given in Eq. (3.1) and Eq. (3.2).

Standard deviation of the mean

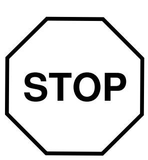
While s tells you how close an *individual measurement* is likely to be to the *true mean*, we would like to know how far the *experimental mean* $\langle x \rangle$ is from the *true mean*. Because $\langle x \rangle$ is an average of N measurements, and is calculated from measurements that lie both above and below the true mean, you might correctly guess that $\langle x \rangle$ is likely to lie closer to the true mean than a typical individual measurement. But how much closer? For large values of N (greater than approximately 10), it turns out that the true means has a 68% chance of lying within $\pm s/\sqrt{N}$ of $\langle x \rangle$. Here N, as in Eq. (3.1), is the number of values used to obtain the mean.

The uncertainty of the mean, given by $s/\sqrt{N}$, is called the *standard deviation of the mean*. This is an accurate name, because $s/\sqrt{N}$ is the best estimate of what you'd get if you measured the mean of N measurements many times and then computed the experimental standard deviation of these means.

So, whenever you quote the uncertainty of a quantity that you've measured N times, you should quote the standard deviation of the mean,

$$\text{standard deviation of the mean} = \frac{s}{\sqrt{N}.} \tag{3.6}$$

13. Determine the standard deviation of the mean in this experiment.
14. In step 11, you were asked to answer the question, "If you were to open one single new M&M bag, what is your prediction for the number of blue M&Ms in this bag?" Now we ask you to answer a different question: "If *everybody* in the class were given a new bag of M&Ms, what is your prediction for the *class average* of the number of blue M&Ms per bag?" Report your answer in the correct format.



Show your estimate to your instructor or TA before continuing.

15. Write a mini-conclusion for Part I. You should include an explanation of the difference between standard deviation and standard deviation of the mean, and when to use each one.

Part II: Does a Pendulum's Period Depend on its Amplitude?

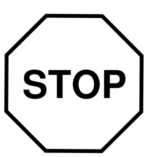
You are provided with a pendulum and a stopwatch accurate to the nearest hundredth of a second. You are to determine whether the period of oscillation of the pendulum (the time for one complete swing) is different for small amplitude swings than it is for large amplitude swings.

Method

Call T_L the measured time for the pendulum to swing through 10 complete swings with large amplitude (about 15°). Similarly, call T_S the measured time for 10 swings of small amplitude (about 5°). The numerical value, $T_L - T_S$, represents one measurement of the difference between the two times. When you measure T_L and T_S repeatedly you should expect to get many different values of $T_L - T_S$. What is the *correct* result for this difference? Is it different from zero? If so, is the difference real or just due to chance? These are some of the questions that can be answered by a statistical analysis of your data, as in Part I.

Procedure

In timing the swings of the pendulum, be as precise as possible. For example, it's better to start timing after the pendulum has swung a couple of times so that timing the start will be exactly like timing the finish. And don't make the mistake of measuring all the values of T_L first and then all the values for T_S. This could introduce a bias due to improvements in your timing with more practice. Finally, consider whether it is better to start and stop at the top of a swing.

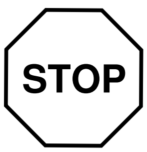


When you have thought about these considerations, come up with a plan for your method of measurement and describe it to your instructor or lab TA. Record your method in your notebook.

1. Use the marks on the wall as starting locations for large swings (about 15° from vertical) and small swings (about 5° from vertical). Then take one measurement of the time T_L for ten large swings and one measurement of the time T_S for ten small swings. Calculate the difference $T_L - T_S$. (You use the time for ten swings rather than just one to reduce the relative effect of uncertainties in starting and stopping the stopwatch.) Continue to alternate measurements of T_L and T_S until you have 12 values of $T_L - T_S$. (Alternation is important

because any improvement due to practice will tend to apply equally to T_L and T_S.)

2. Since you are interested in whether the true mean of $T_L - T_S$ is different from zero, calculate the experimental mean of $T_L - T_S$ and its uncertainty (which? standard deviation or standard deviation of the mean?) For these calculations, use an Excel spreadsheet as in Part I.
3. Write your result for $T_L - T_S$ in standard scientific form.
4. Based on this result, conclude whether T_L is or is not different from T_S, and hence, whether the period of a pendulum depends on the size of its amplitude. Write this conclusion and your reasoning in your lab notebook and discuss it with your lab instructor or TA.



Show your result and conlcusion to your instructor of TA.

Reflection

Please reflect on today's lab in your notebook.

Look back at today's lab-specific objectives (beginning of the lab).

1. How has your understanding of those topics changed through today's lab?
2. How could you explain the difference between "standard deviation" and "standard deviation of the mean" to a peer? When would you advise this peer to use them in lab?
3. In Lab 2, you learned how to propagate uncertainty when more than one measured quantity went into your final result. Comment on how that technique compares to today's process of estimating uncertainty.