Lab 2

Static Equilibrium

Continuing Objectives

3. Be able to write an experimental result (including correct number of significant digits, uncertainty, units).

10. Be able to work with physical vector quantities.

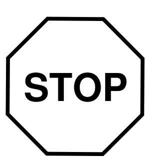
Lab-specific Objectives

1. Find the net force on an object using vector addition.

2. Learn how to propagate uncertainty in experimental measurements to find the total uncertainty on a result.

Introduction

In the previous lab, we dealt with quantities along a straight line, such that the positions and velocities being measured only had one direction: the horizontal direction (or x-component). They could either be positive quantities (moving away from the sensor) or negative quantities (moving toward the sensor), but only along the x-axis.



Suppose we did the same experiment as last week, but instead of walking to and from the sensor, we jumped in place, changing our vertical position over time. Would the sensor record us as changing position? What are the limits to this 1-D view of the world? Is it realistic to use simple 1-D numbers to describe scenarios in physics? Discuss this with your partner and then check in with your TA or instructor.

However, we live in a 3-dimensional world. In order to describe physics in a 3-D space, including forces acting on an object, we need to employ the use of **vectors**.

In the next section, you will have some hands-on practice working with vectors. You have seen vectors in lecture, but vectors might be a new concept, so feel free to ask for help from a TA or instructor as you work through the next section.

Preliminary Procedure

Developing an Excel spreadsheet for vector addition

In the first part of this lab you will add the three force vectors illustrated in Figure 2.1 in two ways. First, you will calculate the vector sum "by hand" using your calculator. Vector addition is something that you need to do repeatedly, so your second task will be to develop an Excel spreadsheet so that a computer will complete the vector addition.

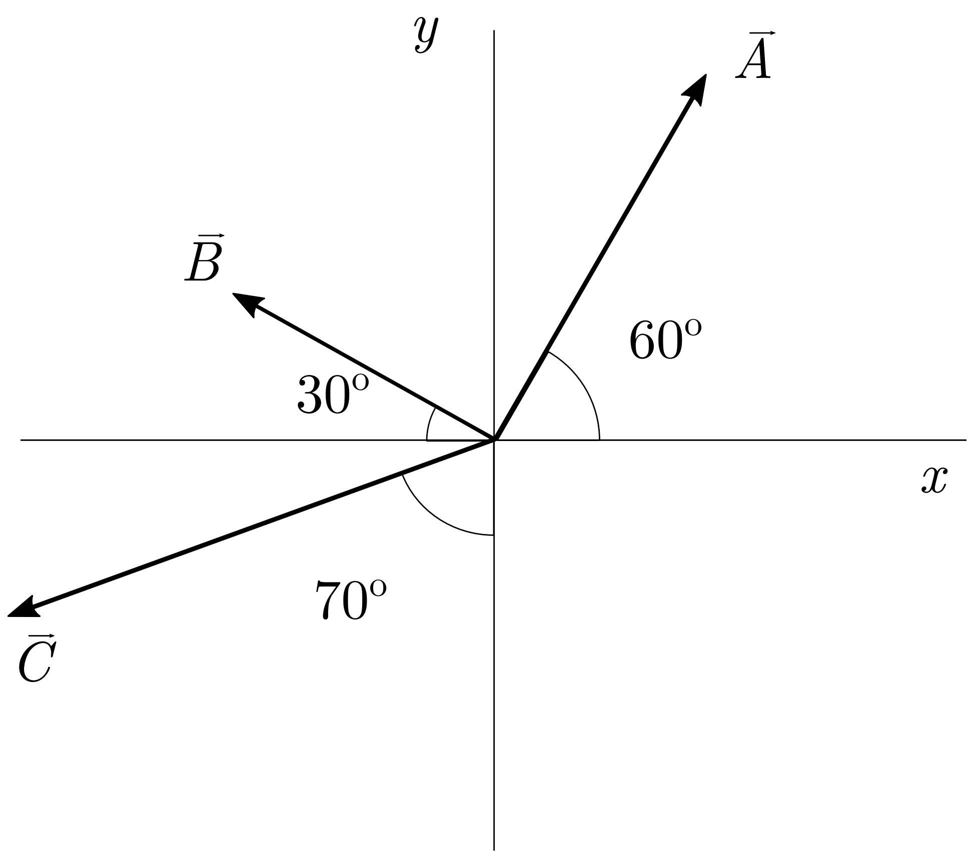


Figure 2.1: Force Diagram of Three Forces

Label	Magnitude (cm)	Angle (°)	Angle (rad)	x-comp.	y-comp.
$\vec{A}$					
$\vec{B}$					
$\vec{C}$					
$\vec{A}+\vec{B}+\vec{C}$					

Table 2.1: Table to Determine Sum of Three Forces

1. Copy Table 2.1 into your lab notebook.
2. **By hand.** Determine the magnitudes of the vectors in Figure 2.1 by measuring their lengths to the nearest half centimeter. Use this information to fill in Table 2.1 and determine the vector sum of the vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$.
3. **Using Excel.** Open the Excel template in the `PHYS 211` folder (inside the `PHYS 211_212` folder on the Desktop). Use this to enter the formulas you used in the table above, i.e., for the last three columns and for the last line. Any formula in Excel must start with "`=`". (For help navigating Excel, refer to your notes from last lab.) Test your Excel spreadsheet by entering the measurements you made in step 2 and seeing if the calculated values Excel returns match what you found by hand.

STOP

Show your Excel spreadsheet to your instructor or TA before continuing.

Experimental Procedure

The apparatus used in this experiment consists of a horizontal force table graduated in degrees around the outer rim and pulleys which may be set at any desired angle. A string passing over each pulley supports a holder upon which weights may be placed. A center pin holds a small ring to which the strings are attached. The ring acts as the "body" on which the forces act. You test if the system is in equilibrium by checking if the ring is not in contact with the center pin.

Question: Why do we say that the ring is in "equilibrium" when it is not in contact with the center pin? What forces will be acting on the ring at that time? What about the *net force*?

Part I: Apparatus setup

1. Set up an arrangement of two forces which do not balance, and record the direction and magnitude of these forces in your lab notebook. (*Choose different weights and angles — do not place the pulleys at* 90° *or* 120° *either relative to one another or as measured on the force table, and use a variety of weights.* On the older force tables, use weights in the range of 200 g – 600 g and on the newer force tables, use weights in the range of 40 g – 100 g.)

2. Call these forces $\vec{A}$ and $\vec{B}$, with magnitudes A and B and directions θ_A and θ_B, respectively. Record the masses M_A and M_B and the angles θ_A and θ_B.

3. Then, by pulling on the third string, find a direction that yields equilibrium. Set the pulley at this position and add weights to the hanger until equilibrium is established. Remember to include the weights of the hangers when recording the forces. Test if the system is in equilibrium by checking that the ring is not in contact with the center pin and centered as nearly as possible about the pin. You may need to gently poke the ring to see if it is truly in static equilibrium. If you can gently poke the ring and it returns to center, then you have found equilibrium.

4. In your lab notebook record for the third force the mass, M_C, and the angle θ_C.

5. Using M_A, M_B, and M_C, determine the magnitudes of the forces (A, B, C).

6. Make two drawings in your notebook:

   (a) One drawing will be a sketch of the force table apparatus, including the positions of the three pulleys, the amount of weight hanging from each pulley, and the strings attached to the center ring.

(b) The second drawing should be a **full page**, scaled *vector addition diagram* of the three forces on the center ring once equilibrium is established. For the length of the vectors, choose an appropriate scale (e.g., 3 cm corresponds to 1 N for older force tables or to 0.4 N for newer force tables, etc.).

HINT: Take a moment to plan out this sketch. Putting the origin at the center of the page is, probably, a bad choice.

STOP Show your work to your instructor or TA before continuing.

In the next section, you will add the three force vectors to calculate the total force. Do you think that they will sum to zero? Discuss this question with your partner and write a short prediction.

Part II: Calculation of total force

Data you will need in your Excel spreadsheet include:

a. Mass (in grams) of each of the three weights corresponding to $\vec{A}$, $\vec{B}$, and $\vec{C}$. On the spreadsheet also fill in the formula for the magnitutes of the forces $\vec{A}$, $\vec{B}$, and $\vec{C}$.

b. Angles made by all three forces with the positive x-axis.

Use your Excel sheet and the formulas you came up with earlier to calculate the x-component of the total force $F_{\text{net},x}$, and the y-component of the total force $F_{\text{net},y}$.

STOP Show your results to your TA or instructor.

At first glance, you might be surprised to find that the net force on the ring when it is in static equilibrium does NOT come out to 0. Does this mean Newton's Laws are wrong?

In any experiment, there will be some **uncertainties** we need to account for and propagate to find the total uncertainty on our final result. *Experimental results are meaningless without a reported uncertainty.* By knowing the uncertainty on a result, we can report the *range* of values our result could reasonably be consistent with and compare that range to the predicted value.

In the next section, we will learn how to find the uncertainties in our measurements and propagate them to find a total uncertainty on our result. We can then compare our result *with associated uncertainty* to the expected value of $\vec{F}_{\text{net}} = 0$ for any object in static equilibrium.

Part III: Propagation of uncertainty

Estimating Uncertainty:

1. Determine the uncertainty in the mass of the third force, M_C, by finding how much weight must be added or removed from the hanger before movement of the ring position is observed.
2. Likewise, determine the uncertainty in angular position of the third force, θ_C, by seeing how many degrees you can move the corresponding pulley to each side before the ring moves off-center.
3. Record the value of each uncertainty (ΔM_C and $\Delta\theta_C$).

We now have uncertainties in the magnitude (via its mass) and angle of the net force $\vec{F}_{\text{net}}$. But how do these uncertainties affect the uncertainty in the components of the total force? These two uncertainties are *uncorrelated;* they don't depend on each other. Furthermore, the components of the total force are not just equal to the magnitude and angles of each of the components - there is a specific mathematical relationship involved.

Because of this, we must propagate the uncertainty in both the magnitude and the angle through our calculations if we want to get an accurate sense for how they affect the uncertainty in the total force.

Note: We will use the same procedure laid out here in almost every lab going forward to determine the uncertainty in our result. Take notes in your lab notebook as you work through today's example.

In the following calculations, we will assume that the uncertainties in the masses and angles of the two forces $\vec{A}$ and $\vec{B}$ are small enough to be negligible. We will consider only the uncertainty in the mass, M_C, and the angle of the third force θ_C to be significant. Be sure to note this assumption in your notebook.

1. Start by choosing ONE variable to focus on, say the uncertainty in the mass, M_C. On the spreadsheet locate the cell containing the mass corresponding to $\vec{C}$. Instead of the original mass, M_C, put into this cell the original mass PLUS the uncertainty in the mass ($M_C + \Delta M_C$).

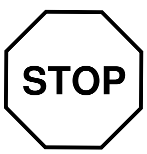
2. When you change the mass of $\vec{C}$, you should notice that all the cells including the componenets of the net force update automatically. Write the new $F_{\text{net},x}$ and $F_{\text{net},y}$ in your lab notebook, labeling them as $F_{\text{net},x}(M_C + \Delta M_C, \theta_C)$ and $F_{\text{net},y}(M_C + \Delta M_C, \theta_C)$. We use this notation because they are the result of the functions $F_{\text{net},x}$ and $F_{\text{net},y}$ for the arguments $M_C + \Delta M_C$ and θ_C. On your EXCEL sheet in the section Part III fill in $F_{\text{net},x}(M_C + \Delta M_C, \theta_C)$ and $F_{\text{net},y}(M_C + \Delta M_C, \theta_C)$ by copying by value. To do so follow the instructions on the spreadsheet on yellow background step 4.

3. To find the extent of the effect of the uncertainty in the mass on our result, find the difference for each component:

$$(\Delta F_{\text{net},x})_{M_C} = F_{\text{net},x}(M_C + \Delta M_C, \theta_C) - F_{\text{net},x}(M_C, \theta_C) \tag{2.1}$$

$$(\Delta F_{\text{net},y})_{M_C} = F_{\text{net},y}(M_C + \Delta M_C, \theta_C) - F_{\text{net},y}(M_C, \theta_C) \tag{2.2}$$

   where $F_{\text{net},x}(M_C, \theta_C)$ and $F_{\text{net},y}(M_C, \theta_C)$ are the values of $F_{\text{net},x}$ and $F_{\text{net},y}$ for the original M_C and θ_C. Add these values and equations to your spreadsheet in the section for Part III. $(\Delta F_{\text{net},x})_{M_C}$ and $(\Delta F_{\text{net},y})_{M_C}$ are the uncertainties in $F_{\text{net},x}$ and $F_{\text{net},y}$ due solely to the uncertainty of the third mass.



Check your progress so far with your instructor or TA. Show them your Excel spreadsheet calculations.

4. On your spreadsheet ensure to change the cell containing the mass corresponding to $\vec{C}$ back to its original value (remove the mass uncertainty). Your values for $F_{\text{net},x}$ and $F_{\text{net},y}$ therefore should be $F_{\text{net},x}(M_C, \theta_C)$ and $F_{\text{net},y}(M_C, \theta_C)$. We will now repeat the above steps for the second variable: the angle θ_C.

5. On your spreadsheet change the cell for the angle of $\vec{C}$ to the angle plus its uncertainty, $\theta_C + \Delta\theta_C$. Again, everthing should update. In your lab notebook and on your spreadsheet write the new $F_{\text{net},x}$ and $F_{\text{net},y}$ that means $F_{\text{net},x}(M_C, \theta_C + \Delta\theta_C)$ and $F_{\text{net},y}(M_C, \theta_C + \Delta\theta_C)$.

6. Find the difference between these new $\vec{F}_{\text{net}}$ components and the original components as you did for the mass uncertainty

$$(\Delta F_{\text{net},x})_{\theta_C} = F_{\text{net},x}(M_C, \theta_C + \Delta\theta_C) - F_{\text{net},x}(M_C, \theta_C) \tag{2.3}$$

   and similarly for $(\Delta F_{\text{net},y})_{\theta_C}$.

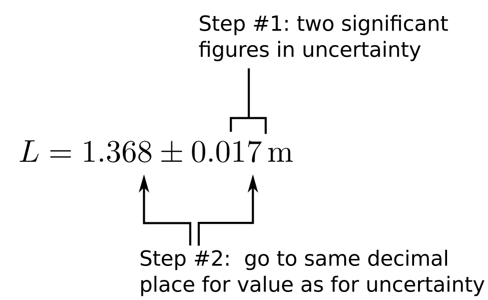
7. We now have two measures of the effect of the uncertainty from each variable. Calculate the overall uncertainty for $F_{\text{net},x}$ using

$$\Delta F_{\text{net},x} = \sqrt{[(\Delta F_{\text{net},x})_{M_C}]^2 + \left[(\Delta F_{\text{net},x})_{\theta_C}\right]^2}. \tag{2.4}$$

   You can do the same calculation for $F_{\text{net},y}$ to find the total uncertainty in that component.

   You have just found the total uncertainty on the $F_{\text{net},x}$ and $F_{\text{net},y}$ components! It may seem like a long process, but with practice, it can become much quicker and easier.

8. Now that we know the total uncertainty on our net force, we can report the result and compare it to the expected result of $\vec{F}_{\text{net}} = 0$. To report a result, use the following format, shown as an example for reporting a length result with associated uncertainty:



   We will use this format in all PHYS 211/212 labs. Use it now to write your $F_{\text{net},x}$ and $F_{\text{net},y}$ results with their associated uncertainty.

9. To evaluate our results, we must see if the expected value of $\vec{F}_{\text{net}} = 0$ falls within the range of our experimental values. We check whether $F_{\text{net},x} = 0$ (and similarly whether $F_{\text{net},y} = 0$.) For reasons that will be more clear in a future lab, we check to see if the predicted value lies within a range of twice the uncertainty of the measured value. This simply means, if we add or subtract $2 \times$ our reported uncertainty to our result, will the expected value of $\vec{F}_{\text{net}} = 0$ be within this range? Test that now separately for $F_{\text{net},x}$ and for $F_{\text{net},y}$ and comment in your lab notebook on your findings. This will form the basis of your conclusion; see below.

Reflection

As part of your conclusion, please reflect on today's lab in your notebook.

Look back at today's lab-specific objectives (beginning of the lab).

1. What activities did you do today that helped practice these objectives?
2. How has your ability to perform these objectives changed through today's lab?
3. Based on your results of measurements for the x- and y-components of the total force on the ring and your uncertainties, **write a conclusion** about whether your data are consistent with Newton's first law applied to an object in static equilibrium.