VPython Class 8: Linear combinations of standing waves

1. Introduction

In the Python Class 7 you made static plots of standing waves at various times. I hope that you got to the point where you could plot things like

- the fundamental mode ($n = 1$):

$$y_1(x,t) = \cos\left(\frac{2\pi}{T}t\right)\,\sin\left(\frac{\pi}{L}x\right),$$

- the first harmonic mode ($n = 2$):

$$y_2(x,t) = \cos\left(\frac{4\pi}{T}t\right)\,\sin\left(\frac{2\pi}{L}x\right),$$

- the linear combination of the $n = 1$ and $n = 2$ modes:

$$Y(x,t) = y_1(x,t) + y_2(x,t).$$

As last time, you can let the length of the string be $L = 1$, and let the speed of the wave be $v = 1$. This implies that the wavelength of the $n = 1$ mode is 2, and the period of this mode is 2 (or equivalently, the frequency of this mode is $f = 1/2$). If you didn't get all of these tasks completed, ask, and we'll help you get them done.

2. Animating plots

Find your notebook from the last Python class and remind yourself how it works. You will need to use pieces of your code in a template of an animation notebook called `animator.ipynb` that I have provided. This notebook makes use of the `matplotlib.animation` module. Look at this template and see if you can recognize familiar pieces.

Your job is to fill in the necessary code in cell labeled **Define functions for plotting**. If you transfer functions correctly from your existing notebook, you should get the expected plot for $t = 0$ after you execute the **Initialize graph for** $t = 0$ cell. If this works, execute the **Start animation** cell.

I would like you to animate the following functions:

1. A linear combination of the two lowest frequency standing wave modes of the string.
2. A linear combination of the $n = 1$, $n = 3$, $n = 5$, and $n = 7$ modes given by

$$Y(x,t) = y_1(x,t) - \frac{1}{9}\,y_3(x,t) + \frac{1}{25}\,y_5(x,t) - \frac{1}{49}\,y_7(x,t). \quad (1)$$

3. A linear combination of modes for odd values of n with many more modes, i.e., the linear combination given by

$$Y(x,t) = \sum_{n=1,3,5,\dots}^{N} \frac{1}{n^2} \sin\left(\frac{n\pi}{2}\right) y_n(x,t), \tag{2}$$

with N, say, as big as 100.

4. The linear combination of modes given by the sum

$$Y(x,t) = \sum_{n=1,3,5,\dots}^{N} \frac{1}{n} \sin\left(\frac{n\pi}{2}\right) \sin\left(\frac{na}{2}\right) y_n(x,t), \tag{3}$$

with $a = 0.4$.

5. Repeat the last combination, but let $a = 0.2$.