UNIV 213 Science & Engineering of Music & Sound Spring 2026

Homework Assignment #4 – due via Moodle at 6:00 pm on Tuesday, Mar. 24, 2026
[G Prob. 4 changed to ungraded 3/23/26]

***Instructions, notes, and hints*:**

Provide the details of all solutions, including important intermediate steps. You will not receive credit if you do not show your work. You may make reasonable assumptions and approximations to compensate for missing information, if any.

The first set of problems will be graded and the rest will not be graded. Only the graded problems must be submitted by the deadline above. Do not submit the ungraded problems.

***Graded Problems*:**

1. The perfect fifth is considered to be the most consonant interval next to the octave. However, many intervals at the low end of the frequency spectrum audible to humans sound dissonant. Using Figs. 5.10 and 8.15 of the textbook (Rossing, Moore, and Wheeler, 3rd ed.), briefly explain why the fifth from C_2 (65.41 Hz) to G_2 (98.00 Hz) sounds much less consonant than the fifth from C_5 (523.25 Hz) to G_5 (783.99 Hz). It is not primarily due to the tuning of the notes to conform to the equally tempered scale; that is, it is not because the frequencies do not have a ratio of exactly 3:2. The difference in consonance would occur even with just intonation.

2. Beats are a linear effect and are most noticeable when two tones that are close in frequency are sounded together. Difference tones are a nonlinear effect and appear at frequencies equal to the differences between integer multiples of the original frequencies of the tones that are sounded. (See Sec. 8.6 of the textbook.) Suppose that two tones with frequencies of 250 Hz and 310 Hz are generated simultaneously by a sound source. Find the frequencies of all possible 2nd-order (quadratic) and 3rd-order (cubic) difference tones that could result.

3. For the just intonation scale, verify by multiplication that raising a note by each of the following pairs of intervals is equivalent to raising the note by an octave:

- **a.** Raising a perfect fifth then raising a perfect fourth
- **b.** Raising a major sixth then raising a minor third

4. **[now an ungraded problem]** Repeat the previous problem for the case of the equally tempered scale. *Hint*: Begin by finding the number of semitones associated with each interval.

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5. Given the frequency ratios for the just intonation scale shown in Fig. 9.5 of the textbook (Rossing, Moore, and Wheeler, 3rd ed.), use arithmetical operations to show that the following intervals are perfect fifths in the key of C, where C is the reference note or tonic. In the intervals listed below, the first note is lower in pitch than the second note. Thus, the intervals in parts **b** and **c** span two octaves:

   a. E : B
   b. G : D
   c. A : E

6. Find the frequency ratio of the interval D : A in the just intonation scale, where the first note (D) is lower in pitch than the second note (A). *Hint*: The interval is close to a perfect fifth, but the frequency ratio is not exactly 3:2 (1.5000). This illustrates one of the problems with just intonation.

Ungraded Problems:

The following problems will not be graded, but you should attempt to solve them on your own and then check the solutions. Do not give up too quickly if you struggle with one or more of them. Move on to a different problem and then return to the difficult one after a few hours.

1. A sound effects technician at a movie studio wants to create a sound consisting of two tones with a wavering amplitude that corresponds to four beats per second. One of the tones will have a frequency of exactly 200 Hz. The other tone will have a frequency close to (but not exactly equal to) 500 Hz. Find the required frequency of the second tone to achieve four beats per second.

2. Piano tuning can be accomplished by counting the number of beats that are generated by difference tones when two notes are played simultaneously. (Note that the beats heard in this case are the result of the nonlinear intermodulation of the two notes. The beating occurs between harmonics that are nearly the same frequency.) Suppose that a 440 Hz tuning fork confirms that the A_4 note on a certain piano has the correct pitch in the equally tempered scale. The E_5 note should have the frequency 659.25 Hz (660 Hz would be a perfect fifth).

   a. How many beats per second should the tuner hear when the E_5 note is correctly tuned? (In practice, the tuner would probably count the number of beats in 10 sec and then divide by 10 to obtain the number of beats/sec.)
   b. The same number of beats/sec would be heard if the E_5 note were accidentally tuned to 660.75 Hz, which is too high. The tuner could determine whether the frequency is correct by slightly increasing the tension on the E_5 strings. Why?

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3. The notes in the western chromatic scale (the one that includes all sharps and flats, 12 notes per octave) are:

C $C^\sharp$ D $D^\sharp$ E F $F^\sharp$ G $G^\sharp$ A $A^\sharp$ B C

The "♯" symbol indicates a sharp, which raises the note by a half step. In the equally tempered scale, a note raised by a sharp is enharmonic with (has the same pitch as) the next higher note lowered by a flat, indicated by the "♭" symbol. Thus, $D^\sharp$ is the same as $E^\flat$. By international agreement, the standard frequency of the note A_4 (the A above middle C and near the middle of a piano keyboard) is 440 Hz. Use arithmetical operations to find the frequencies associated with the following notes in the equally tempered scale. You may check your answers using Table 9.2 of the textbook:

- **a.** D_1
- **b.** $E_3^\flat$
- **c.** $G_5^\sharp$
- **d.** B_7