ENGR 4/695 Advanced Topics in Engineering Mathematics Fall 2026

Lecture Outline for Friday (1:00 pm lecture), Sept. 18, 2026

1. One application of eigenvalues and eigenvectors: More efficient matrix multiplication

    a. Suppose that you know the eigenvalues of eigenvectors of $N \times N$ matrix A (they have been precalculated). To multiply a known vector $\mathbf{v}$ by A, first express $\mathbf{v}$ as a linear combination of the eigenvectors $\{\mathbf{x}_i\}$ so that

    $$\mathbf{v} = \sum_{i=1}^{N} a_i \mathbf{x}_i$$

    Multiplying $\mathbf{v}$ by A is equivalent to

    $$A\mathbf{v} = A\left(\sum_{i=1}^{N} a_i \mathbf{x}_i\right) = \sum_{i=1}^{N} a_i A \mathbf{x}_i = \sum_{i=1}^{N} a_i \lambda_i \mathbf{x}_i$$

2. Eigensystem insights and expectations

    a. $A\mathbf{x}_i = \lambda \mathbf{x}_i \quad \rightarrow \quad AX = X\Lambda$
    b. Eigenvalues can repeat, and one or more can be zero.
    c. Eigenvectors associated with $\lambda = 0$ can be non-zero.
    d. An eigenvector multiplied by a scalar constant is still an eigenvector: $A(k\mathbf{x}_i) = \lambda(k\mathbf{x}_i) \quad \rightarrow \quad kA\mathbf{x}_i = k\lambda \mathbf{x}_i \quad \rightarrow \quad A\mathbf{x}_i = \lambda \mathbf{x}_i$
    e. $A\mathbf{x}_i = \lambda \mathbf{x}_i \quad \rightarrow \quad (A - \lambda I)\mathbf{x}_i = \mathbf{0}$ can be used to find eigenvalues and eigenvectors.

3. Some basic theorems

    a. If A is square with real entries, then any complex eigenvalues/eigenvectors come in conjugate pairs.
    b. If A is square, then 0 is an eigenvalue iff (if and only if) A is singular.
    c. $\det(A) = \lambda_1 \lambda_2 \lambda_3 \ldots \lambda_N$
    d. If A is nonsingular and λ is an eigenvalue, then $1/\lambda$ is an eigenvalue of A^{-1}; both eigenvalues have the same corresponding eigenvectors.
    e. Eigenvalues of upper/lower-triangular and diagonal matrices are the diagonal values.