ECEG 390 Theory and Applications of Electromagnetics Spring 2026

Homework Assignment #6 – due via Moodle at 8:00 pm on Monday, Mar. 30, 2026
[Graded Prob. 4 deferred to HW #7]

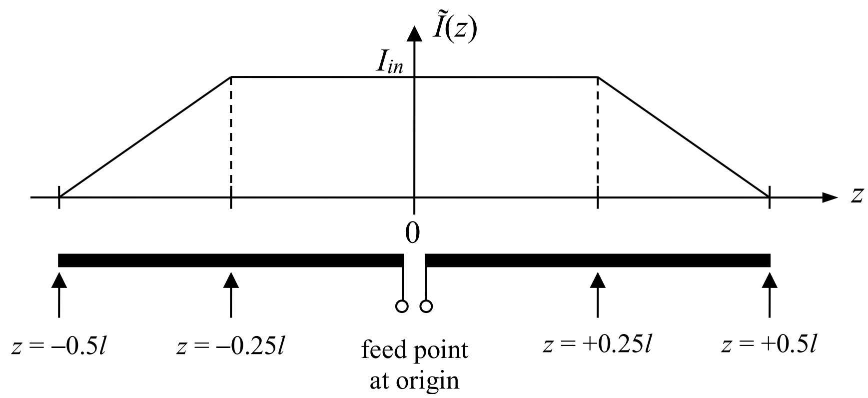
***Instructions, notes, and hints*:**

For full credit, provide the details of all solutions, including important intermediate steps. You may make reasonable assumptions and approximations to compensate for missing information, if any. If your answers differ from the posted answers but you justify any approximations that you make, you will be given full credit.

Note that the first set of problems will be graded and the rest will not be graded. Only the graded problems must be submitted by the deadline above. Do not submit the ungraded problems.

***Graded Problems*:**

1. An electrically short ($l << \lambda$) center-fed dipole antenna operating at 10 MHz has an unusual design feature that causes the current distribution along its length to have the functional dependence shown below. The input current at the feed point is $\tilde{I}_{in}$. The current magnitude is constant over the middle half of the antenna, and it tapers linearly to zero at the ends over the outer sections. Derive an expression for the far electric field radiated by the antenna, and then determine the directivity of the antenna. The current does not experience a phase shift along the antenna because it is a standing wave; therefore, $\tilde{I}_{in}$ can be assumed to have a real value, and $\tilde{I}(z)$ has a constant phase of 0° everywhere along the antenna. (That is, the current is a real-valued function of z.) The tiny gap at the feed point is infinitesimally narrow. The antenna is made from aluminum rod stock with a diameter of 3.18 mm (about 1/8 in).



2. For the antenna in the previous problem, derive an expression for the radiation resistance as a function of the length l. Obtain a numerical value for $l = 0.02\lambda$, and compare it to the radiation resistance of a standard short dipole (with a triangular current distribution) and that of a Hertzian dipole (uniform distribution) of the same length.

3. For the antenna in the previous two problems, derive an expression for loss resistance R_{loss} as a function of the length l. Obtain numerical values for R_{loss} and the efficiency for $l = 0.02\lambda$, and compare them to those of a short dipole (triangular current distribution) and a Hertzian dipole (uniform distribution) of the same length. The input current at the feed point is $\tilde{I}_{in}$.

(*continued on next page*)

4. **[Deferred to HW #7]** A highly advanced reflector antenna has been built out of a new conductive polymer. The physical aperture of the antenna is 67 cm in diameter, but because of "spillover" from the feed antenna, the aperture efficiency is only 60%. Although the antenna presents a 50 Ω load to the 50 Ω transmission line feeding it, measurements indicate that 10 Ω of the 50 Ω total is due to power loss from the limited conductivity of the polymer, the type of feed arrangement used, and various losses in the feed antenna. (Note that aperture efficiency and power efficiency are different effects and can exist simultaneously.) The antenna is to be used in an 8.0 GHz communication link between two facilities that are 20 km apart. The facilities are identical, and each one has one of the new antennas and a transmitter that can produce 5.0 W of output power; however, the 40 m long transmission line that connects the transmitters and receivers to each antenna each introduce 6.0 dB of loss. (The antenna can be switched between transmission and reception mode.) Find the power density in nW/m^2 in the vicinity of the antenna at one of the facilities when the other is sending a signal. Neglect atmospheric attenuation and reflections from the ground and other objects between the facilities.

5. An AM broadcast station has a transmitter with an output power of 10 kW and operates at a frequency of 980 kHz. The antenna is a ground-mounted monopole with an input impedance of $120 + j240\ \Omega$, but an impedance matching network is used to couple the antenna to the 50 Ω coaxial cable that brings the signal from the transmitter to the antenna. The monopole antenna radiates uniformly in azimuth (but not in elevation) and has a directivity of approximately 5.5 dBi. The ground beneath the antenna is effectively part of it, but because an extensive system of radials has been installed at the base of the antenna, the equivalent loss resistance is approximately 5.0 Ω. Assuming that the losses in the coaxial cable and impedance matching network are negligible, find the power density (magnitude of the Poynting vector) at a distance of 7.0 km from the antenna.

6. A highly directive antenna with a power efficiency of 90% radiates a rotationally symmetric main lobe with a half-power beamwidth (HPBW) of 5.8°. (The half-power beamwidth is the range of angles over which the radiated power density is within 3 dB of the maximum value.) Assuming that that power density is negligibly small outside the HPBW, estimate the gain of the antenna in dBi.

***Ungraded Problems*:**

The following problems will not be graded, but you should attempt to solve them on your own and then check the solutions. Do not give up too quickly if you struggle with one or more of them. Move on to a different problem and then come back to the difficult one after a few hours.

1. An amateur radio operator has just received her license and is excited to get on the air on the 40-meter band (operating frequency of around 7.1 MHz). Her first antenna is a half-wave dipole antenna made from #12 copper wire (diameter of 2.053 mm). Assuming that the current distribution is a perfect half sinusoid and ignoring the effects of the ground (i.e., assuming operation in free space), find the efficiency and gain of the antenna. You may use mathematical analysis software to evaluate the resulting integral if you wish.

(*continued on next page*)

2. Find the effective aperture in terms of square wavelengths (λ^2) for the following antennas. Also, find the radius in terms of λ of the circular area that is equivalent to the effective aperture, and compare the radius to the size of the antenna.

a. standard center-fed half-wave dipole
b. center-fed dipole of length 0.1λ
c. Yagi-Uda array with a total length of 3.83λ, and a directivity of 15.8 dBi

3. Horn antennas are analogous to the bells on brass and woodwind musical instruments, and they are often used in antenna measurement facilities since they are structurally rugged, relatively easy to use, and maintain their calibration very well. One type of horn antenna is made from a flared waveguide that ends in a rectangular opening (aperture). With a carefully calibrated gain specification, it can be used as a gain standard in antenna test ranges. The commercially available antenna depicted below (diagram taken from the datasheet) has a 42 mm × 37 mm aperture and a gain of 10.9 dBi at 10.0 GHz. Assuming that the power efficiency of the antenna is 90%, find its aperture efficiency.