ECEG 390 Theory and Applications of Electromagnetics Spring 2026

Homework Assignment #5 – due via Moodle at 11:59 pm on Monday, Mar. 23, 2026

***Instructions, notes, and hints*:**

For full credit, provide the details of all solutions, including important intermediate steps.

You may make reasonable assumptions and approximations to compensate for missing information, if any. If your answers differ from the posted answers but you justify any approximations that you make, you will be given full credit.

The constitutive parameters (ε, μ, and σ) of many important engineering materials are available in Appendix B of the textbook (Ulaby and Ravaioli, 8th ed.).

Note that the first set of problems will be graded and the rest will not be graded. Only the graded problems must be submitted by the deadline above. Do not submit the ungraded problems.

***Graded Problems*:**

1. A 0.01λ-long Hertzian dipole is located at the origin of a coordinate system, is oriented along the z-axis, and operates at a frequency of 10 MHz. The input current I_0 is $10 \angle 0°$ A. Evaluate the R and θ-components of the phasor electric field (near field) radiated by the antenna in the $\theta = 30°$ direction at the distances a) $R = 1\lambda$ and b) $R = 10\lambda$. You may use Matlab, Mathematica, or some other computational software to solve this problem. If you do, include a print-out of your script or program session to submit with your homework. Also consider showing some intermediate calculations in case your answer is incorrect; it might help you receive some partial credit.

2. Evaluate the far electric field approximation for the Hertzian dipole described in the previous problem in the $\theta = 30°$ direction at the distances a) $R = 1\lambda$ and b) $R = 10\lambda$. The antenna length, frequency, and input current are the same. One of these distances is not in the far field, but evaluate both cases anyway using the far-field expression. Compare your results to parts a and b of the previous problem, and explain why the values that you obtained for each part are or are not close to each other. You must evaluate the far-field results using nothing more than a calculator; do not use mathematical software except to check your answers.

3. A terrestrial (land-based) data link system uses a transmitter connected to a Yagi-Uda antenna that has a gain of 16 dBi. Measurements reveal that the received signal's power level is –137 dBm. (The dBm unit represents power referenced to 1 mW; in this case, –137 dBm = 0.0200 fW, or 2.00×10^{-17} W.) However, if the data is to be detected reliably, the signal that arrives at the receiver's input terminals must have a minimum power level of –133 dBm (0.0501 fW). The received power level is proportional to the radiated signal's power density in the vicinity of the receiving antenna. A transmitter with a higher output power would be too expensive, so the design team decides to replace the original transmitting antenna with another one with higher gain. Find the minimum required gain (in dBi) of the new antenna.

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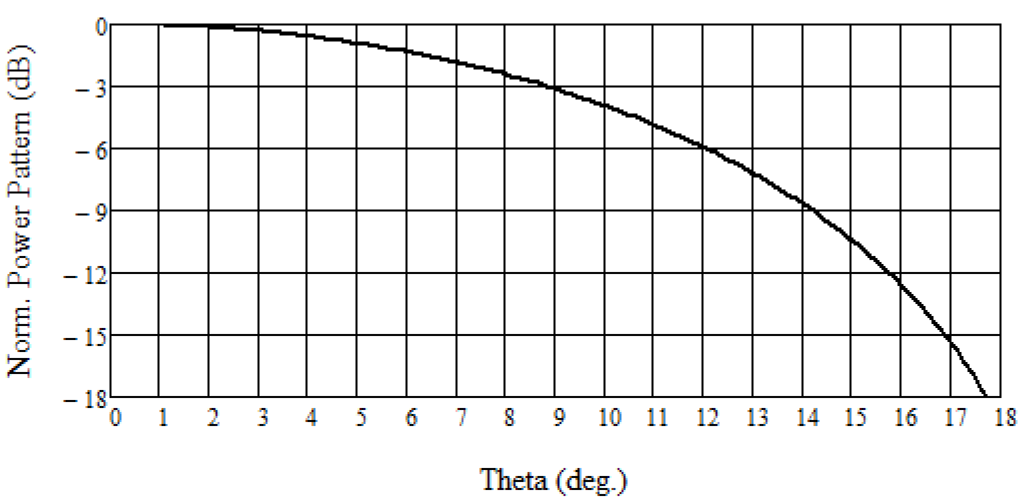
4. Suppose that the normalized power pattern of a certain antenna is given by the expression below. Confirm that $F(\theta, \phi)$ has a maximum value of one over all values of θ and ϕ, and then show that the directivity of the antenna is 6.0 (7.8 dBi) using analytical (i.e., manual or "on-paper") evaluation. You may check your answer using Matlab, Mathematica, or your calculator to perform the integration numerically, but you must evaluate the integral analytically to receive full credit. You may use an integral table.

$$F(\theta, \phi) = \begin{cases} \left|\sin^2\theta \cos^2\phi\right|, & 0 \le \theta \le 180°, -90° \le \phi \le 90° \\ 0, & 0° \le \theta \le 180°, 90° \le \phi \le 270° \end{cases}$$

5. Sketch the *unnormalized* radiation pattern of the antenna described in the previous problem in the *xy*-plane; that is, plot the actual directivity vs. angle in degrees. Choose an appropriate angle to plot against, and use a decibel scale on the vertical axis ranging from −30 dBi to 10 dBi. You must sketch the pattern by hand (as neatly as possible, of course), but you may check your answer using mathematical software or a graphing calculator. Check your plot at a few key angles to ensure that it has a reasonably accurate shape.

6. A microwave link antenna has the *normalized* power pattern $F(\theta, \phi)$ shown below. The pattern is rotationally symmetric about the *z*-axis; that is, there is no ϕ-dependence. Note, however, that the pattern plot is in terms of dB relative to the maximum value of $F(\theta, \phi)$, not as a multiplying factor. Negligible power is radiated for $\theta > 20°$. The maximum directivity of the antenna is 21.3 dBi (in the $\theta = 0°$ direction), so the figure below gives the directivity relative to 21.3 dBi. For example, the directivity in the $\theta = 9°$ direction is 21.3 dBi – 3.0 dB = 18.3 dBi. The antenna's efficiency is 80%, the transmitter feeding the antenna has an output power of 20 W, and the transmission line between the transmitter and antenna is essentially lossless. The receiving station is 35 km away, and the system operates at a frequency of 12 GHz. A violent storm forces the antenna out of its proper orientation so that the receiving site is 15° away from the direction of maximum gain. Find the power density (in W/m^2) produced in the vicinity of the receiving station.



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***Ungraded Problems*:**

The following problems will not be graded, but you should attempt to solve them on your own and then check the solutions. Do not give up too quickly if you struggle with one or more of them. Move on to a different problem and then come back to the difficult one after a few hours.

1. The differential form of Ampère's law in the time domain is given below. Use dimensional analysis to show that all three terms in the equation have the same units. *Hint*: The vector curl operation is essentially a linear combination of spatial derivatives.

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}.$$

2. For each of the following media, explain whether it should be considered "source-free" or "source filled" in the macroscopic sense. Remember that a source can be either a current (**J**) or charge (ρ_v) distribution. Note that individual static charges in the immediate vicinity of equal and opposite charges are not considered sources. For example, a water molecule has a negative charge center on the oxygen side and two positive charge centers on the hydrogen side; however, the total numbers of electrons and protons in the molecule are equal and the resulting electric field is tiny in extent. Non-ionized water is therefore considered source-free in the macroscopic sense even though individual molecules can be considered sources (static electric dipoles) in the microscopic sense. This problem is intended to be a thought exercise.

- **a.** the sun
- **b.** the dielectric layer of a printed circuit board
- **c.** the copper part of household electrical wiring
- **d.** aluminum lightning rod as a thunderstorm is approaching
- **e.** aluminum lightning rod on a nice day with high humidity
- **f.** cup of distilled water
- **g.** cup of salt water
- **h.** copper traces that constitute a computer's RAM address bus – power on
- **i.** copper traces that constitute a computer's RAM address bus – power off

3. Equations (9.8b) and (9.8c) in the textbook (Ulaby and Ravaioli, 7th ed. and 8th ed.) give the R and θ-components, respectively, of the electric field radiated by a Hertzian dipole centered at the origin of the coordinate system and oriented along the z-axis. The expressions are shown below. In the far-field approximation, the terms involving $(kR)^2$ and $(kR)^3$ are considered to be insignificant in magnitude and therefore are ignored. Determine the distance R in wavelengths that an observer must be from a Hertzian dipole so that the relative magnitudes of the $1/(kR)^2$ and $1/(kR)^3$ terms are 1.0% or less than that of the $1/kR$ term.

$$\widetilde{E}_R = \frac{2I_o l k^2}{4\pi}\eta_o e^{-jkR}\left[\frac{1}{(kR)^2} - \frac{j}{(kR)^3}\right]\cos\theta$$

$$\widetilde{E}_\theta = \frac{I_o l k^2}{4\pi}\eta_o e^{-jkR}\left[\frac{j}{kR} + \frac{1}{(kR)^2} - \frac{j}{(kR)^3}\right]\sin\theta.$$

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4. Starting with the expressions given in Equations 9.9a and 9.9b in the textbook for the far field of a Hertzian dipole, find the *time-domain* electric and magnetic fields [that is, $\mathbf{E}(R, \theta, \phi, t)$ and $\mathbf{H}(R, \theta, \phi, t)$] at the points listed below. The dipole has the following characteristics: peak input current = 12 A; length = 1.0 m; frequency = 1.0 MHz; surrounded by free space; centered on the coordinate system origin; oriented along the z-axis.
   a. at $(R, \theta, \phi) = (10{,}000, 30°, 0°)$
   b. at $(R, \theta, \phi) = (10{,}075, 30°, 0°)$

   Compare the magnitude and phase obtained at each point, and comment on the significance of the comparison.

5. If a Hertzian dipole is aligned along the z-axis in the Cartesian coordinate system, then maximum radiation in the far field occurs in all directions in the $\theta = 90°$, $\phi = 0°$ to $360°$ plane (i.e., the plane perpendicular to the antenna) in the spherical coordinate system. Find the values of θ and ϕ that define the two cones that form the boundary between where the far-field radiated power density is less than and greater than 10% of its maximum value at a given distance R from the antenna.

6. A large class of reflector antennas have normalized power patterns that can be approximated by the sinc function, where sinc(x) = sin(x)/x. Consider a reflector with a pattern described by the expression given below

$$F(\theta) = \begin{cases} \left|\dfrac{\sin(10\theta)}{10\theta}\right|^2, & \theta \le \dfrac{\pi}{10} \\ 0, & \text{elsewhere} \end{cases}$$

   Note that the pattern is rotationally symmetric about the z-axis, so it is not a function of ϕ. Find the direction of maximum radiation, and estimate or calculate the directivity (in dBi) of the antenna using a calculator or software to perform numerical integration. (You may use one method to check the other; this approach is recommended.) Be careful with the special case at $\theta = 0$; consider applying L'Hospital's differentiation rule at that angle.