ECEG 390 Theory and Applications of Electromagnetics Spring 2026

Homework Assignment #3 – due via Moodle at 11:59 pm on Monday, Feb. 23, 2026

***Instructions, notes, and hints*:**

Provide the details of all solutions, including important intermediate steps. You will not receive credit if you do not show your work.

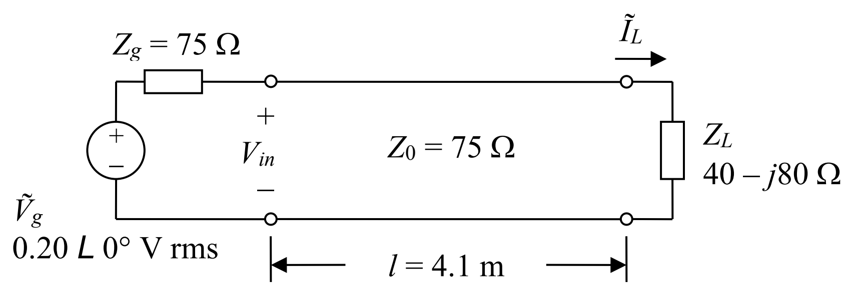
You may make reasonable assumptions and approximations to compensate for missing information, if any. In those cases, your answers could differ significantly from the posted answers. If you justify any approximations that you make, you will be given full credit.

The constitutive parameters (ε, μ, and σ) of many important engineering materials are available in Appendix B of the textbook (Ulaby and Ravaioli, 8th ed.).

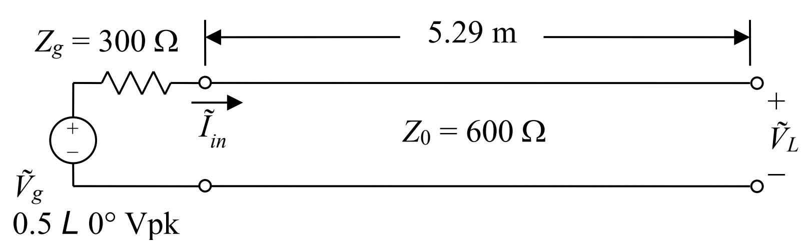
Note that the first set of problems will be graded and the rest will not be graded. Only the graded problems must be submitted by the deadline above. Do not submit the ungraded problems.

***Graded Problems*:**

1. A 4.1-m section of 75 Ω coaxial line with polyethylene insulation is driven by a signal source as shown in the figure below. The generator (signal source) voltage expressed as a phasor is $\tilde{V}_g = 0.20 \angle 0°$ V rms, and the load impedance is $Z_L = 40 - j80\ \Omega$. Find the phasor input voltage $\tilde{V}_{in}$ and the phasor load current $\tilde{I}_L$. Express both phasors in polar form. The operating frequency is 300 MHz.



2. Find the phasor representation (in polar form) of the load voltage $\tilde{V}_L$ at the location of the open circuit at the far end of the transmission line stub shown below. Also find the phasor input current $\tilde{I}_{in}$ of the stub. Express both quantities in polar form. The operating frequency is 10 MHz, and the line is a parallel-wire type with air insulation.



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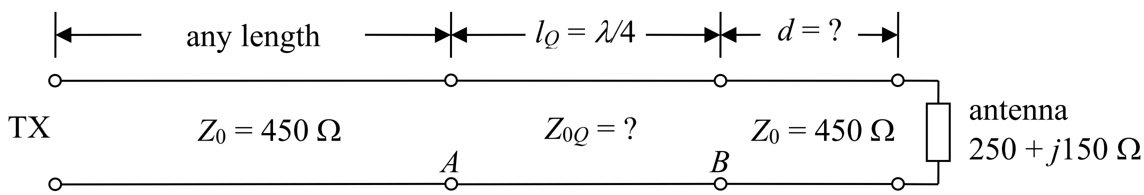
3. Design a quarter-wave section (i.e., find distance the d and the quarter-wave section's characteristic impedance Z_{0Q}) to match an antenna with an impedance of $250 + j150\ \Omega$ at an operating frequency of 20 MHz to a 450 Ω parallel-wire air-insulated transmission line. The matching system is depicted schematically below. The distance d from the antenna to the load side of the quarter-wave section must be as short as possible. Also, find the required physical length l_Q of the quarter-wave section in meters. "TX" is a standard abbreviation for transmitter; it represents the signal source at the input end.



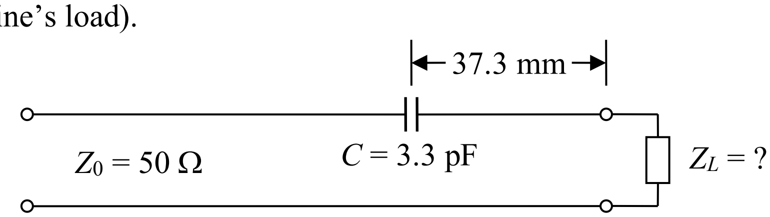
4. For the matching system considered in the previous problem, find:

- **a.** the VSWR along the line between the transmitter and the input of the quarter-wave section (point A).
- **b.** the VSWR along the quarter-wave section.
- **c.** the VSWR between the output of the quarter-wave section (point B) and the load.
- **d.** The required separation distance between the wires in the quarter-wave section if #18 AWG wire is used. See the "American wire gauge" link on the Web Links page at the course web site.

5. For the matching system considered in the previous two problems, suppose that a capacitor with a reactance of $-150\ \Omega$ at the operating frequency is inserted in series with the antenna. The quarter-wave matching section is then redesigned. Find the new required distance d, quarter-wave section length l_Q, and characteristic impedance Z_{0Q}.

6. A 50 Ω microstrip line will be used to supply a wireless broadband signal to an amplifier with an input impedance of $30 + j45\ \Omega$ and that operates at a frequency of 2.45 GHz. The effective relative permittivity of the microstrip line is 4.2. Find the closest location to the load (i.e., the point nearest the amplifier's input terminals) at which an SMT (surface mount technology) capacitor can be inserted in series with the line to achieve an impedance match. Also find the required capacitance value in picofarads.

7. Suppose that you are working for a company that has found an old piece of equipment with the microstrip matching system depicted schematically below. The load Z_L is an antenna, but you don't know its input impedance. However, you can measure the dimensions of the microstrip line, and you can read the value of the surface mount capacitor C. You also know that the system operated at 1.6 GHz and that the microstrip line has an effective relative permittivity of $\varepsilon_r = 4.0$. Use the given information to estimate the input impedance of the antenna (the line's load).



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Ungraded Problems:

The following problems will not be graded, but you should attempt to solve them on your own and then check the solutions. Do not give up too quickly if you struggle with one or more of them. Move on to a different problem and then come back to the difficult one after a few hours.

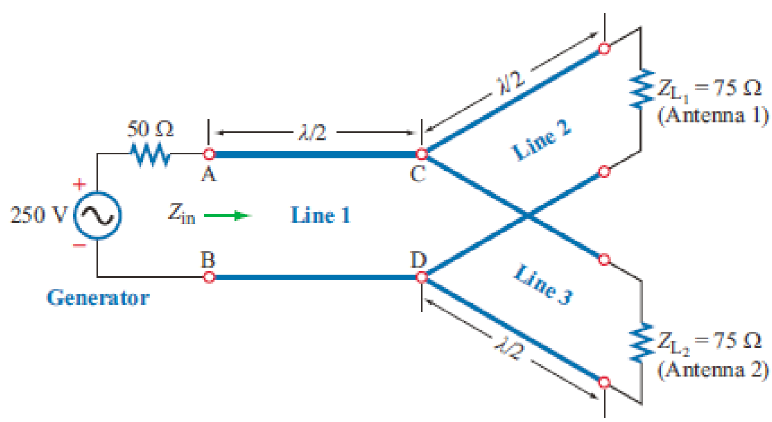
1. Attempt to design a series-element matching system to achieve an impedance match between a load of $Z_L = j40\ \Omega$ and a transmission line with $Z_0 = 75\ \Omega$. The dimension of the line section may be in wavelengths. Explain conceptually why this task is impossible.

2. Attempt to design a quarter-wave line section to achieve an impedance match between a load of $Z_L = j40\ \Omega$ and a transmission line with $Z_0 = 75\ \Omega$. The line and stub dimensions may be in wavelengths. Explain conceptually why this task is impossible.

3. Use the expression given below for the input impedance of a loaded transmission line to show that any line with a purely reactive load (i.e., $Z_L = jX$, where X can be a capacitive or inductive reactance) must have an input impedance of zero at those locations $z_{\min}$ where voltage minima occur. You may use one of the expressions for $d_{\min}$ given in the textbook.

$$Z_{in} = Z_0 \frac{1+\Gamma e^{j2\beta z}}{1-\Gamma e^{j2\beta z}} = Z_0 \frac{1+|\Gamma| e^{j(\theta_r+2\beta z)}}{1-|\Gamma| e^{j(\theta_r+2\beta z)}}$$

4. As shown below, two antennas are being fed by a single signal source. Line 1 has a characteristic impedance of 50 Ω, and lines 2 and 3 each have characteristic impedances of 75 Ω. Find the VSWR along each of the line sections (i.e., along lines 1, 2, and 3). Also find the input impedance Z_{in} of the whole antenna system at junction A-B.



Source: F. T. Ulaby and U. Ravaioli, *Fundamentals of Applied Electromagnetics*, 7th ed., Pearson Education, Inc., Upper Saddle River, NJ, 2015.

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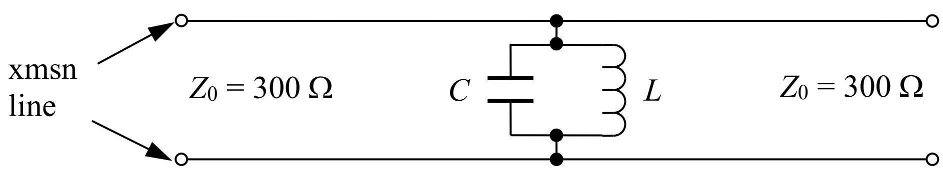
5. Imagine a capacitor and an inductor connected across a transmission line at the same point, as shown below. The components have reactances of equal magnitude but opposite algebraic sign at the operating frequency, so they form a parallel resonant circuit that has an equivalent combined impedance of infinity. That means that the presence of the circuit is not "felt" by waves propagating along the line at the resonant frequency. However, waves at other frequencies will experience partial reflection at the location of the LC circuit, with greater reflection at frequencies farther from resonance. The line and LC circuit therefore form a type of band-pass filter. Although lumped capacitors and inductors can be used, more typically transmission line stubs are employed. Find the electrical lengths (i.e., in wavelengths) of a pair of shunt (parallel) stubs necessary to create reactances of

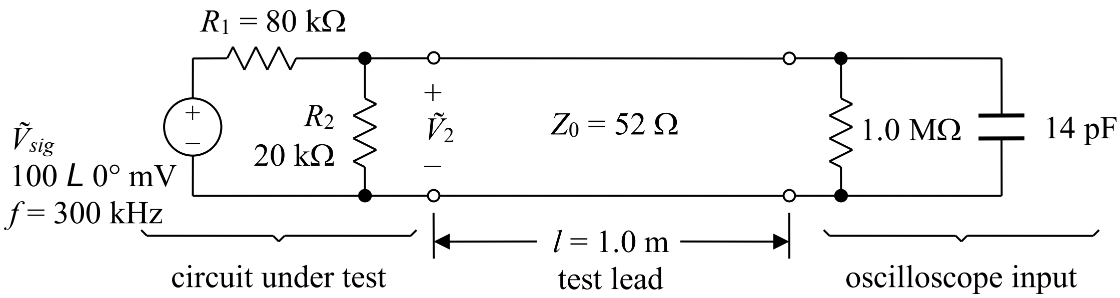
a. $X_L = |X_C| = 30\ \Omega$
b. $X_L = |X_C| = 300\ \Omega$,

where X_L is the equivalent inductive input reactance of one stub, and X_C is the equivalent capacitive input reactance of the other stub. Use open-circuited stubs. Both stubs are connected in parallel with the line at the same point (where the inductor and capacitor are connected in the diagram below).

For each case, add the lengths of the two stubs together. What is interesting about the sum? What would the stub structure look like if $X_L = |X_C| \to \infty$?



6. Suppose that you have been asked to measure the voltage across a voltage divider that operates at 300 kHz. The figure below depicts the signal source $\tilde{V}_{sig}$ and the divider formed by R_1 and R_2. The circuit is electrically small (i.e., it is less than 0.01λ in size). To make the measurements, you are using an oscilloscope with a test lead made from a 1.0 m length of RG-58A coaxial cable ($Z_0 = 52\ \Omega$ with polyethylene dielectric). As shown in the figure, the oscilloscope's input port can be modeled as a resistance of 1.0 MΩ in a parallel with a capacitance of 14 pF. (These values are printed next to the input ports of most oscilloscopes, including the ones in the Bucknell ECE labs.) Find the phasor voltage $\tilde{V}_2$ (in polar form) that would be measured across the 20 kΩ resistor with the test lead connected across the resistor as shown. Compare your answer to the voltage that would appear across R_2 if the leads were not connected. Do the cable and oscilloscope load down the circuit significantly?



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7. A load of roughly $150 + j50\ \Omega$ at an operating frequency of 1.6 GHz is connected to a 50 Ω microstrip line with $\varepsilon_r = 4.0$. Because of manufacturing issues, the load is likely to vary considerably from the indicated value. An important goal is to make all parts of the system as small as possible. The experienced designers know that the closest point to an inductive load where a series matching element can be placed is likely to have a negative input reactance (X_{in}) and therefore would require an inductor to achieve a match. They would like to use a variable inductor to accommodate the variable load impedance, but inductors are bulky, expensive, and often lossy in the RF range. The designers can implement a variable inductance using a fixed inductor in series with a variable capacitor as shown below, but the only available variable capacitor range is 1–10 pF. (The drawing is not to scale. The size of the LC combination is much less than l_{main}.) Find the required fixed inductance value L so that the range of adjustability of the LC combination is centered on the required inductive reactance value. Also find the required distance l_{main} from the load in millimeters.