ECEG 390 Theory and Applications of Electromagnetics Spring 2025

Final Exam General Information

The following policies will be in effect for the exam. They will be included in a list of instructions and policies on the first page of the exam:

1. You will be allowed to use a non-wireless enabled calculator, such as a TI-99. Please make sure that you know how to use all of the relevant features of your calculator, especially those related to complex numbers and the representation of phase. Lack of proficiency with your calculator that leads to incorrect solutions will not be considered an extenuating circumstance. If you do not know how to use a particular feature of your calculator, then you must complete the calculations in question manually. Assistance with the operation of your calculator cannot be provided during the exam.
2. You will be allowed to use up to **five** 8.5 × 11-inch two-sided handwritten help sheets. No photocopied material or copied and pasted text or images are allowed. If there is a table or image from the textbook or some other source that you feel would be helpful during the exam and that is not included on the table and formula sheets I will provide, please notify me.
3. All help sheets will be collected at the end of the exam but will be returned to you later if you wish to have them back.
4. If you begin the exam after the start time, you must complete it in the remaining allotted time. However, you may not take the exam if you arrive after the first student has completed it and left the room. The latter case is equivalent to missing the exam.
5. **You may not leave the exam room before completing your exam without prior permission except in an emergency or for an urgent medical condition. Please use the restroom before the exam.** If you have a medical condition that might require you to leave the room, you must notify me before the exam begins. Only one student at a time may be absent from the room and must leave any electronic devices in the room.

The final exam will take place **3:30–6:30 pm on Wednesday, May 14 in Dana 319**.

As stated in the ECEG 390 Course Policies & Information sheet (the syllabus), the final exam will consist of two parts. The first part will resemble the four in-semester exams and, like them, will be quantitative in nature. It will focus primarily on the material covered since Exam #4, but you will also need to have a good grasp of the material from earlier in the semester. The second part (the Final Concept Exam) will be mostly qualitative and will assess your understanding of the key course concepts. The two parts will be designed to take a total of approximately 1.5 hours to complete, but you may use the full three hours if necessary.

Your graded final exam will not be returned to you, nor will the solutions be posted. However, you may make an appointment with me at any time to review your final exam and discuss your performance on it. I will keep your final exam at least until you graduate from Bucknell.

A list of topics to be covered on the exam begins on the next page.

Review Topics for Final Exam – revised 5/11/25

The following is a list of topics that could appear in one form or another on the exam. Not all of these topics will be covered, and it is possible that an exam problem could cover a detail not specifically listed here. However, this list has been made as comprehensive as possible. **You should be familiar with the topics on the review sheets for all of the previous exams in addition to those listed below.**

Although significant effort has been made to ensure that there are no errors in this review sheet, some might nevertheless appear. The textbook and the supplemental readings are the final authority in all factual matters, unless errors have been specifically identified there. You are ultimately responsible for obtaining accurate and authoritative information when preparing for the exam.

Polarization loss factor (PLF)

- more general form of Friis transmission formula that incorporates various losses

  $$P_{RX} = P_{TX} G_{TX} G_{RX} \left(\frac{\lambda}{4\pi R} \right)^2 F_{TX}\left(\theta_{TX}, \varphi_{TX}\right) F_{RX}\left(\theta_{RX}, \varphi_{RX}\right) L_{TX} L_{RX} L_{path} L_{pol},$$

  where:
  $F_{TX}(\theta_{TX}, \phi_{TX})$ = normalized power pattern of TX antenna
  $F_{RX}(\theta_{RX}, \phi_{RX})$ = normalized power pattern of RX antenna
  L_{TX} = losses in transmitting system (transmission line, matching network, etc.)
  L_{RX} = losses in receiving system (transmission line, matching network, etc.)
  L_{path} = path loss due to attenuation in medium between antennas
  L_{pol} = polarization mismatch loss
- polarization loss factor (L_{pol})

  $$\text{PLF} = \left| \hat{\mathbf{e}}_{TX} \cdot \hat{\mathbf{e}}_{RX}^* \right|^2,$$

  where
  $\hat{\mathbf{e}}_{TX}$ = normalized electric field of transmit antenna
  $\hat{\mathbf{e}}_{RX}$ = normalized electric field of receive antenna if it is used in transmit mode
- procedure for finding normalized electric field
- Antennas and other signal sources are rarely designed to produce elliptical polarization intentionally, but it is highly likely that a linearly polarized or CP wave will become elliptically polarized to some extent after reflection, refraction, diffraction, and other interactions with objects in a real environment.

Maxwell's equations in differential time-domain form (the "point-wise" equations):

Faraday's law:	$\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$	Ampére's law:	$\nabla \times \mathbf{H} = \mathbf{J} + \dfrac{\partial \mathbf{D}}{\partial t}$
Gauss's law:	$\nabla \cdot \mathbf{D} = \rho_v$	"Magnetic Gauss's" law:	$\nabla \cdot \mathbf{B} = 0$

Maxwell's equations in differential time-harmonic (phasor) form:

Gauss's law:	$\nabla \cdot \tilde{\mathbf{D}} = \tilde{\rho}_v$	"Magnetic Gauss's" law:	$\nabla \cdot \tilde{\mathbf{B}} = 0$
Faraday's law:	$\nabla \times \tilde{\mathbf{E}} = -j\omega \tilde{\mathbf{B}}$	Ampére's law:	$\nabla \times \tilde{\mathbf{H}} = \tilde{\mathbf{J}} + j\omega \tilde{\mathbf{D}}$

Constitutive relations (valid in time-domain form or time-harmonic, i.e., phasor, form), but all assume that the constitutive parameters are constant scalars (isotropic, nondispersive media):

- $\mathbf{D} = \varepsilon\mathbf{E}$, where ε is the permittivity of the medium
- $\mathbf{B} = \mu\mathbf{H}$, where μ is the permeability of the medium
- $\mathbf{J} = \sigma\mathbf{E}$, where σ is the conductivity of the medium

Source-free vs. source-filled regions (i.e., are $\mathbf{J}$ and/or ρ_v zero or non-zero?)

Plane-wave propagation in lossy media

- wave equations for time-harmonic fields in a source-free $\left(\tilde{\mathbf{J}} = 0,\ \tilde{\rho}_v = 0\right)$, lossy region
  - $\nabla^2\tilde{\mathbf{E}} - \gamma^2\tilde{\mathbf{E}} = 0$
  - $\nabla^2\tilde{\mathbf{H}} - \gamma^2\tilde{\mathbf{H}} = 0$
  - $\gamma^2 = -\omega^2\mu\varepsilon_c = -\omega^2\mu\left(\varepsilon' - j\varepsilon''\right)$
- time domain forms of fields: $\mathbf{E}(t) = \mathrm{Re}\left\{\tilde{\mathbf{E}}e^{j\omega t}\right\}$ and $\mathbf{H}(t) = \mathrm{Re}\left\{\tilde{\mathbf{H}}e^{j\omega t}\right\}$
- complex permittivity
  - definition: $\varepsilon_c = \varepsilon - j\dfrac{\sigma}{\omega} = \varepsilon' - j\varepsilon''$, where $\varepsilon' = \varepsilon$ and $\varepsilon'' = \dfrac{\sigma}{\omega}$
  - allows source-free Ampére's law to be rewritten as

    $$\nabla\times\tilde{\mathbf{H}} = j\omega\tilde{\mathbf{D}} + \tilde{\mathbf{J}} = j\omega\varepsilon\tilde{\mathbf{E}} + \sigma\tilde{\mathbf{E}} = j\omega\left(\varepsilon - j\frac{\sigma}{\omega}\right)\tilde{\mathbf{E}} = j\omega\left(\varepsilon' - j\varepsilon''\right)\tilde{\mathbf{E}}$$
  - loss tangent is a convenient way to classify electromagnetic behavior of materials

    $$\frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\varepsilon\omega}$$
- complex permeability is also possible but is not commonly encountered except for ferrous (iron-containing) materials or those containing cobalt or nickel

Uniform plane waves in lossy media

- "uniform:" there is no variation in field strength in directions normal (transverse) to prop. direction (e.g., if prop. is in z-direction, then $\partial/\partial x$ and $\partial/\partial y = 0$)
- "plane:" planar (flat) phase fronts, rather than spherical or cylindrical (or other shape)
- TEM (transverse electromagnetic) waves, planar or not:
  - **E** and **H** are both perpendicular to the dir. of prop. and to each other
  - neither **E** nor **H** has a component in the dir. of prop.
  - waves can be planar, cylindrical, spherical, or can conform to other kinds of nonstandard orthogonal coordinate systems, such as ellipsoidal
- for a TEM plane wave that has only an E_x component and is propagating in the $\pm z$-direction:
  - lossless medium: $\dfrac{\partial^2 E_x}{\partial z^2} + k^2 E_x = 0$ (similar eqn for H field)
  - lossy medium: $\dfrac{\partial^2 E_x}{\partial z^2} - \gamma^2 E_x = 0$ (similar eqn for H field)
- solutions:
  - lossless medium: $\tilde{\mathbf{E}} = \hat{\mathbf{x}}\tilde{E}_{xo}e^{\pm jkz}$ (similar eqn for H field)
  - lossy medium: $\tilde{\mathbf{E}} = \hat{\mathbf{x}}\tilde{E}_{xo}e^{\pm\alpha z}e^{\pm j\beta z}$ (similar eqn for H field)
- sign of jkz (or $j\beta z$) indicates direction of propagation

- for TEM waves:

  $\tilde{\mathbf{E}} = -\eta_c \hat{\mathbf{k}} \times \tilde{\mathbf{H}}$ and $\tilde{\mathbf{H}} = \frac{1}{\eta_c} \hat{\mathbf{k}} \times \tilde{\mathbf{E}}$,

  $\hat{\mathbf{k}}$ is unit vector in dir. of prop.; η_c is defined below
- $k = \beta = 2\pi/\lambda$, but pay attention to context
- general expressions for propagation constant and intrinsic impedance:
  - complex propagation constant

    $\gamma^2 = -\omega^2 \mu \varepsilon_c = -\omega^2 \mu \left(\varepsilon' - j\varepsilon''\right)$ and $\gamma = \alpha + j\beta$
  - attenuation constant (in Np/m): $\alpha = \omega \left\{ \frac{\mu\varepsilon'}{2} \left[\sqrt{1 + \left(\frac{\varepsilon''}{\varepsilon'} \right)^2} - 1 \right] \right\}^{1/2}$
  - phase constant (in rad/m): $\beta = \omega \left\{ \frac{\mu\varepsilon'}{2} \left[\sqrt{1 + \left(\frac{\varepsilon''}{\varepsilon'} \right)^2} + 1 \right] \right\}^{1/2}$
  - $\eta_c = \sqrt{\frac{\mu}{\varepsilon' - j\varepsilon''}} = \sqrt{\frac{\mu}{\varepsilon'}} \left(1 - j\frac{\varepsilon''}{\varepsilon'} \right)^{-1/2}$ (assumes no sig. magnetic loss; i.e., $\mu'' = 0$)
- low-loss dielectrics: $\frac{\varepsilon''}{\varepsilon'} << 1$ (assumes no significant magnetic loss; i.e., $\mu'' = 0$)
  - attenuation constant (in Np/m): $\alpha \approx \frac{\omega\varepsilon''}{2}\sqrt{\frac{\mu}{\varepsilon'}} = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$
  - phase constant (in rad/m): $\beta \approx \omega\sqrt{\mu\varepsilon}$
  - $\eta_c \approx \sqrt{\frac{\mu}{\varepsilon}}$ (approximately real)
  - $\eta_0 = 120\pi\ \Omega = 377\ \Omega$ in free space (vacuum or air)
- good conductors: $\frac{\varepsilon''}{\varepsilon'} >> 1$ (assumes no significant magnetic loss)
  - attenuation constant (in Np/m): $\alpha \approx \omega\sqrt{\frac{\mu\varepsilon''}{2}} = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\pi f \mu\sigma}$
  - phase constant (in rad/m): $\beta \approx \sqrt{\pi f \mu \sigma}$
  - $\eta_c \approx (1+j)\sqrt{\frac{\pi f \mu}{\sigma}} = (1+j)\frac{\alpha}{\sigma}$
- quasi-conductors): neither $\frac{\varepsilon''}{\varepsilon'} << 1$ nor $\frac{\varepsilon''}{\varepsilon'} >> 1$; must use general formula above
- permeability is almost never complex, except in the case of lossy ferromagnetic materials such as iron, cobalt, nickel, and their alloys
- relationships between period T, frequency f (or ω), wavelength λ, and phase constant k (also called the wave number) of waves (or α and β); expressions below apply to all media:

  $T = \frac{1}{f}$ $\qquad k = \beta = \frac{2\pi}{\lambda}$ $\qquad u_p = \frac{\omega}{\beta}$ $\qquad \delta_s = \frac{1}{\alpha}$

- **E** and **H** are *in* phase if medium is lossless
- **E** and **H** are *out of* phase if medium is lossy (although never more than 45° for isotropic materials with constant ε, μ, and σ)
- speed of wave found from $\frac{\partial(\omega t - kz + \phi)}{\partial t} = 0$ or $\frac{\partial(\omega t - \beta z + \phi)}{\partial t}$; $u_p = \frac{\partial z}{\partial t}$, ϕ is const.
- perfect plane waves cannot exist in nature, but they are very good approximations of spherical waves (and other practical types of waves) over regions of limited size at large distances from wave sources

Electromagnetic power density

- instantaneous Poynting vector (a time-dependent quantity):

  $\mathbf{S}(t) = \mathbf{E}(t) \times \mathbf{H}(t)$ (unit is W/m^2)

  instantaneous power per unit area of a wave (general formula for any kind of time variation)
- time-average Poynting vector (a time-independent quantity) represents the time-average value of **S** and is applicable to time-harmonic (periodic) waves:

  $\mathbf{S}_{av} = \frac{1}{2}\mathrm{Re}\left\{\tilde{\mathbf{E}} \times \tilde{\mathbf{H}}^*\right\}$,

  where E and H are expressed in terms of their peak (not rms) values
- lossless media: $\mathbf{S}_{av} = \hat{\mathbf{k}}\frac{\left|\tilde{E}_1\right|^2 + \left|\tilde{E}_2\right|^2}{2\eta}$,

  where $\tilde{E}_1$ and $\tilde{E}_2$ are the two complex orthogonal vector components of the wave perpendicular to the direction of propagation and are expressed in peak units
- lossy media: $\mathbf{S}_{av} = \hat{\mathbf{k}}\frac{\left|\tilde{E}_1\right|^2 + \left|\tilde{E}_2\right|^2}{2|\eta_c|}e^{-2\alpha z}\cos\theta_\eta$, where $\eta_c = |\eta_c|e^{j\theta_\eta}$
- attenuation through the atmosphere increases substantially in millimeter wave range (20 GHz and above, primarily because of resonances of gas molecules)
- α[dB/m] = 8.686 α[Np/m]
- total power in wave intercepted by a surface of area A: $P_{av} = \iint_A \mathbf{S}_{av} \cdot d\mathbf{s}$

  If wave is incident normal to aperture and *uniform* over aperture: $P_{av} = |\mathbf{S}_{av}|A$

  If $\mathbf{S}_{av}$ is not uniform over aperture, then evaluation of integral is more complicated because $P_{av} \neq |\mathbf{S}_{av}|A$.

  Surface A could be the effective aperture of an antenna.
  If antenna is sensitive to only one vector component of wave, then the received power is proportional to the fraction of the total wave power contained in that component.
  If antenna has polarization completely orthogonal to polarization of wave, $P_{av} = 0$.
  See polarization loss factor (PLF) section above.

skin depth & skin effect

- electric and magnetic field decay as $e^{-\alpha z}$ in lossy medium (where z is the coordinate along which wave propagates)
- skin depth is the distance at which wave has decayed by e^{-1} relative to starting value, so

  $\delta_s = \frac{1}{\alpha}$

- for good conductors: $\delta_s = \frac{1}{\omega}\sqrt{\frac{2}{\mu\varepsilon''}} = \sqrt{\frac{2}{\omega\mu\sigma}} = \frac{1}{\sqrt{\pi f \mu\sigma}}$
- skin effect
  - tendency of high-frequency currents to flow near the surface of conductor because of skin depth; current magnitude decays exponentially with depth into conductor
  - not relevant at DC (skin depth is infinite)
  - not significant at low frequencies where wire diameter << λ

Boundary conditions (See Table 6-2) **– NOT COVERED ON SPRING 2025 EXAM**

- Tangential electric field intensity: $E_{1\tan} = E_{2\tan}$
- Normal electric flux density: $D_{1n} - D_{2n} = \rho_s$ (surface charge density is infinitely thin)
- Tangential magnetic field intensity: $H_{1\tan} - H_{2\tan} = J_s$ (surface current density is infinitely thin and flows in direction orthogonal to magnetic field; $J_s \neq 0$ only if one medium is a conductor)
- Normal magnetic flux density: $B_{1n} = B_{2n}$
- Inside a perfect conductor, the normal and tangential fields are all zero.

Wave reflection and transmission at planar interfaces **– NOT COVERED ON SP 25 EXAM**

- for normal incidence, all components of **E** and **H** waves are tangential to interface
- distinction between incident, reflected, and transmitted waves
- actual, total field on incident side of boundary is the sum of incident and reflected waves (which propagate in opposite directions)
- at the boundary only: $\mathbf{E}^i + \mathbf{E}^r = \mathbf{E}^t$ and $\mathbf{H}^i + \mathbf{H}^r = \mathbf{H}^t$ (because all wave components are tangential to interface)
- definitions of reflection and transmission coefficients:

  $\Gamma = \frac{\tilde{E}_0^r}{\tilde{E}_0^i}$ and $\tau = \frac{\tilde{E}_0^t}{\tilde{E}_0^i}$
- expressions for Γ and τ in terms of material properties are found by applying BCs
- reflection coefficient: $\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$, where material no. 1 is on the incident side of the boundary; applies to both lossless and lossy media
- transmission coefficient: $\tau = \frac{2\eta_2}{\eta_2 + \eta_1}$; applies to both lossless and lossy media
- $1 + \Gamma = \tau$; Γ and τ can be complex, but $|\Gamma| < 1$, and $|\tau| < 2$
- solution of reflection/transmission problems for normal incidence is very similar to that for transmission line problems
- power densities of incident, reflected, and transmitted waves (when medium 1 and medium 2 are both lossless):

  $\tilde{\mathbf{S}}_{av}^i = \hat{\mathbf{k}}\frac{\left|\tilde{E}_0^i\right|^2}{2\eta_1}$, $\tilde{\mathbf{S}}_{av}^r = -\hat{\mathbf{k}}\left|\Gamma\right|^2\frac{\left|\tilde{E}_0^i\right|^2}{2\eta_1}$, and $\tilde{\mathbf{S}}_{av}^t = \hat{\mathbf{k}}\left|\tau\right|^2\frac{\left|\tilde{E}_0^i\right|^2}{2\eta_2}$
- It is possible for the transmitted (medium 2) field to have a greater magnitude than the incident (medium 1) field, but power is conserved because, in that case, $|\eta_1| < |\eta_2|$.
- for lossless or lossy media, $\frac{1-\left|\Gamma\right|^2}{\left|\eta_{c1}\right|}\cos\theta_{\eta 1} = \frac{\left|\tau\right|^2}{\left|\eta_{c2}\right|}\cos\theta_{\eta 2}$

Relevant course material:

HW: #9
Reading: Assignments from April 21 through May 5, including the supplemental reading "Polarization Loss Factor"

Two supplemental sheets with Tables 2-1, 2-2, 2-4, 3-1, 3-2, and 7-1 from the textbook (Ulaby and Ravaioli, 8th ed.) plus several fundamental formulas will be made available to you during the exam. The supplement sheets are available at the ECEG 390 course Moodle site. In addition, up to **five** 8.5 × 11-inch two-sided handwritten help sheets may be used during the exam.

This exam will focus primarily on the course outcomes listed below and related topics:

5. Mathematically express and/or analyze the polarization of an electromagnetic wave. [focus is on polarization loss factor]
6. Relate the attenuation, wavelength, and/or speed of a TEM wave propagating through a lossy medium to the medium's known constitutive parameters.
7. Predict the magnitudes and propagation directions of reflected and transmitted plane waves at a planar interface between two materials. [normal incidence only]

The course outcomes are listed on the Course Policies and Information sheet, which was distributed at the beginning of the semester and is available on the Syllabus and Policies page at the course web site. The outcomes are also listed on the Course Description page. Note, however, that some topics not directly related to the course outcomes could be covered on the exam as well.

Note that we covered Outcome #7 only minimally before the semester ended. That topic will therefore constitute only a small portion of the Final Exam.

Preparation for the Final Concept Exam

The Final Concept Exam, which constitutes 5% of the overall course grade, will assess your understanding of the major overarching ideas covered in the course. In one sense, it is difficult to suggest *preparation* strategies since the best preparation is engagement with the course material throughout the semester, and that time is now past.

However, there are two approaches that might help you solidify your understanding of the material. The first is to go over the review sheets for this exam and the prior ones carefully and make sure that you understand the basic principles encompassed in each topic. The second approach is to review the "Concepts" sections at the end of each chapter in the textbook that we covered. Remember that several supplemental readings were also assigned, so you ought to skim (or read for the first time?) that material as well. One other approach that might help is to read the introductory material at the beginning of each textbook chapter that we covered.

The Final Concept Exam will likely consist of 10–15 questions in a combination of formats such as true-false statements, multiple choice, or short written answers. A few very short (one line) calculations or interpretations of formulas might also be required. If you are well prepared, the concept exam should not take long to complete.