ECEG 390 Theory and Applications of Electromagnetics Spring 2025

Policies and Review Topics for Exam #4

The following policies will be in effect for the exam. They will be included in a list of instructions and policies on the first page of the exam:

1. You will be allowed to use a non-wireless enabled calculator, such as a TI-89. Please make sure that you know how to use all of the relevant features of your calculator, especially those related to complex numbers and the representation of phase. Lack of proficiency with your calculator that leads to incorrect solutions will not be considered an extenuating circumstance. If you do not know how to use a particular feature of your calculator, then you must complete the calculations in question manually. Assistance with the operation of your calculator cannot be provided during the exam.
2. You will be allowed to use up to four 8.5 × 11-inch two-sided handwritten help sheets. No photocopied material or copied and pasted text or images are allowed. If there is a table or image from the textbook or some other source that you feel would be helpful during the exam and that is not included on the table and formula sheets that I will provide, please notify me.
3. All help sheets will be collected at the end of the exam but will be returned to you either immediately or soon after the exam.
4. If you begin the exam after the start time, you must complete it in the remaining allotted time. However, you may not take the exam if you arrive after the first student has completed it and left the room. The latter case is equivalent to missing the exam.
5. **You may not leave the exam room before completing your exam without prior permission except in an emergency or for an urgent medical condition. Please use the restroom before the exam.**

The exam will begin at 12:30 pm on Friday, April 25 in Dana 319. You will have until 1:50 pm to complete the exam.

The following is a list of topics that could appear in one form or another on the exam. Not all of these topics will be covered, and it is possible that an exam problem could cover a detail not specifically listed here. However, this list has been made as comprehensive as possible. **You should be familiar with the topics on the review sheets for the previous exams as well.**

Although significant effort has been made to ensure that there are no errors in this review sheet, some might nevertheless appear. The textbook is the final authority in all factual matters, unless errors have been specifically identified there. You are ultimately responsible for obtaining accurate information when preparing for the exam.

Faraday's law

- time domain form: $\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$; phasor form: $\nabla \times \tilde{\mathbf{E}} = -j\omega \tilde{\mathbf{B}}$
- integrate over open surface and apply Stokes' theorem to obtain integral forms (time-domain and phasor-domain cases shown below):

  $V_{emf} = \oint_C \mathbf{E} \cdot d\mathbf{l} = -N \iint_S \dfrac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s}$ and $V_{emf} = \oint_C \tilde{\mathbf{E}} \cdot d\mathbf{l} = -j\omega N \iint_S \tilde{\mathbf{B}} \cdot d\mathbf{s}$; N = no. of turns

- sum of voltage drops around contour C is equal to the surface integral of time rate of change of magnetic flux density *crossing* surface. If there are N closely parallel loops, then there are effectively N surfaces.
- right-hand rule relates direction of contour C (and therefore the direction of integration of the contour integral) to surface normal $\hat{\mathbf{n}}$
- polarity of V_{emf}: direction of C points from + to − at the terminals; with that polarity, V_{emf} has a *negative* value if **B** points in direction of surface normal and is increasing.
- A loop with voltage and current induced in it acts like a source from the perspective of any external circuit that might be connected to it.

Lenz's law

- direction of current induced in loop is such that the **B** field that it creates opposes the change in the external **B** field that produced the current
- satisfies law of conservation of energy

Small loop antennas (circumference << λ)

- often used in portable devices operating at "low" frequencies ~VHF/UHF and below
- **B** field must have a nonzero component that is normal to the loop in order to induce voltage at terminals
- radiation pattern null in the directions normal to plane of loop
- for a small transmitting loop of area A in xy-plane and centered at origin, far field is $\tilde{\mathbf{E}} = \hat{\boldsymbol{\varphi}} \dfrac{\eta k^2 A\tilde{I}}{4\pi} \dfrac{e^{-jkR}}{R} \sin\theta$, where $\tilde{I}$ is peak (not rms) value of input current; A = loop area; η = intrinsic impedance of surrounding medium; $k = 2\pi/\lambda$. Loop area can be any shape, but circular area maximizes area for a given conductor length (greatest efficiency).
- radiation resistance of N-turn loop of area A and radius r in free space

  $$R_{rad} = 31{,}200 N^2 \left(\frac{A}{\lambda^2}\right)^2 = 308{,}000 N^2 \left(\frac{r}{\lambda}\right)^4$$

- loss resistance of loop with total wire length l and wire radius a is

  $R_{loss} = R_{ohmic} = \dfrac{l}{2\pi a} R_s$, where $R_s = \sqrt{\dfrac{\pi f \mu_c}{\sigma_c}}$ (lower-case L in numerator, not "1")

- inductive reactance of loop is usually tuned out by addition of a capacitor
- electrically large loops (i.e., > 0.1λ or so in diameter) have very different radiation patterns compared to those of small loops; significant radiation normal to loop; 1λ circumference loop has a radiation peak normal to the loop

Effective aperture (or effective area)

- relationship to gain or directivity

  $A_e = \dfrac{\lambda^2 D}{4\pi} = \dfrac{\lambda^2}{4\pi}\dfrac{G}{\xi}$ or $D = \dfrac{4\pi}{\lambda^2} A_e \quad \rightarrow \quad G = \xi \dfrac{4\pi}{\lambda^2} A_e$, where ξ = power efficiency

- effective aperture related to physical aperture size (area) A_{ap} by aperture efficiency (ξ_{ap}); ξ_{ap} is not the same as ξ

  $$A_e = \xi_{ap} A_{ap}$$

- effective aperture is not equal to physical aperture; the aperture sizes are comparable only for electrically large antennas like reflectors; the comparison to physical size is meaningless for electrically small antennas, especially ones for which D = 1.5

- beware of mixing multiplying factors and values in dBi in formulas for D and G; also remember to convert ξ expressed in % to fractional value; e.g., if G = 3 dBi, substitute 2, not 3, for G in formula.

Friis transmission formula

- used for "link budget" analysis
- assuming antennas are oriented in directions of max. radiation, are co-polarized, there are no reflections from nearby objects, there is no loss in the space between the antennas, and there are no losses along connecting transmission lines, then

  $$P_{RX} = P_{TX} G_{TX} G_{RX} \left(\frac{\lambda}{4\pi R} \right)^2 = P_{TX} \frac{\xi_{TX} \xi_{RX} A_{eTX} A_{eRX}}{\lambda^2 R^2},$$

  where TX refers to the transmitter and RX to the receiver
- if antenna(s) are not correctly oriented, then G (or D) for each antenna must be reduced by normalized power pattern value in direction of TX and/or RX:

  $$P_{RX} = P_{TX} G_{TX} G_{RX} \left(\frac{\lambda}{4\pi R} \right)^2 F_{TX}\left(\theta_{TX}, \varphi_{TX}\right) F_{RX}\left(\theta_{RX}, \varphi_{RX}\right)$$
- transmission line loss:
  - reduces transmitted and/or received power
  - should not be included in efficiency; it is more properly treated separately because the transmission line is not part of the antenna
  - Loss has a negative dB value but is often expressed as a positive value in common usage; pay attention to context; negative dB value corresponds to loss factor < 1 (e.g., loss = 8 dB corresponds to a "gain" of −8 dB and a multiplying factor of $10^{-8/10} = 0.1585$)

Maxwell's equations in differential time-domain form (the "point-wise" equations):

- Gauss's law: $\nabla \cdot \mathbf{D} = \rho_v$
- Faraday's law: $\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$
- "Magnetic Gauss's" law: $\nabla \cdot \mathbf{B} = 0$
- Ampére's law: $\nabla \times \mathbf{H} = \mathbf{J} + \dfrac{\partial \mathbf{D}}{\partial t}$

Maxwell's equations in differential time-harmonic (phasor) form:

- Gauss's law: $\nabla \cdot \tilde{\mathbf{D}} = \tilde{\rho}_v$
- Faraday's law: $\nabla \times \tilde{\mathbf{E}} = -j\omega\tilde{\mathbf{B}}$
- "Magnetic Gauss's" law: $\nabla \cdot \tilde{\mathbf{B}} = 0$
- Ampére's law: $\nabla \times \tilde{\mathbf{H}} = \tilde{\mathbf{J}} + j\omega\tilde{\mathbf{D}}$

  where tilde (~) over variable indicates that it is a phasor

Constitutive relations (valid in time-domain form or time-harmonic, i.e., phasor, form), but all assume that the constitutive parameters are constant scalars (isotropic, nondispersive media):

- $\mathbf{D} = \varepsilon\mathbf{E}$, where ε is the permittivity of the medium
- $\mathbf{B} = \mu\mathbf{H}$, where μ is the permeability of the medium
- $\mathbf{J} = \sigma\mathbf{E}$, where σ is the conductivity of the medium

Source-free vs. source-filled regions (i.e., are $\mathbf{J}$ and/or ρ_v zero or non-zero?)

Can calculate electric (magnetic) field from magnetic (electric) field via:

- $\tilde{\mathbf{E}} = \dfrac{1}{j\omega\varepsilon}\nabla\times\tilde{\mathbf{H}}$ (source-free Ampére's law)
- $\tilde{\mathbf{H}} = \dfrac{-1}{j\omega\mu}\nabla\times\tilde{\mathbf{E}}$ (source-free Faraday's law)

Wave equations for time-harmonic fields in a source-free $\left(\tilde{\mathbf{J}}=0,\ \tilde{\rho}_v=0\right)$, lossless ($\varepsilon$ and μ are both real) region:

- $\nabla^2\tilde{\mathbf{E}} + k^2\tilde{\mathbf{E}} = 0$
- $\nabla^2\tilde{\mathbf{H}} + k^2\tilde{\mathbf{H}} = 0$
- $k^2 = \omega^2\mu\varepsilon$

Time domain forms of fields: $\mathbf{E}(t) = \mathrm{Re}\left\{\tilde{\mathbf{E}}e^{j\omega t}\right\}$ and $\mathbf{H}(t) = \mathrm{Re}\left\{\tilde{\mathbf{H}}e^{j\omega t}\right\}$

Uniform plane waves

- "uniform:" there is no variation in field strength in directions normal (transverse) to prop. direction (e.g., if prop. is in z-direction, then $\partial/\partial x$ and $\partial/\partial y = 0$)
- "plane:" planar (flat) phase fronts, rather than spherical or cylindrical (or other shape)
- perfect plane waves do not exist but are very good approximations of spherical waves (and other practical types of waves) over regions of limited size far from wave sources
- TEM (transverse electromagnetic) waves, planar or not:
  - **E** and **H** are both perpendicular to the dir. of prop. and are perpendicular to each other; neither **E** nor **H** has a component in the dir. of prop.
  - waves can be planar, cylindrical, spherical, or conforming to other kinds of nonstandard orthogonal coordinate systems, such as ellipsoidal
- for a TEM plane wave that has only an E_x component and is propagating in the $\pm z$-direction:
  - lossless medium: $\dfrac{\partial^2 E_x}{\partial z^2} + k^2 E_x = 0$ (similar eqn for H field)
  - solution: $\tilde{\mathbf{E}} = \hat{\mathbf{x}}\tilde{E}_{xo}e^{\pm jkz}$ (similar eqn for H field)
  - sign of jkz exponent indicates direction of propagation
- for TEM waves in lossless media:
  - $\tilde{\mathbf{E}} = -\eta\hat{\mathbf{k}}\times\tilde{\mathbf{H}}$
  - $\tilde{\mathbf{H}} = \dfrac{1}{\eta}\hat{\mathbf{k}}\times\tilde{\mathbf{E}}$

    where $\hat{\mathbf{k}}$ is unit vector in dir. of prop.
- can use Faraday's law to find **H** from **E** or source-free Ampère's law to find **E** from **H**
- $k = \beta = 2\pi/\lambda$, but pay attention to context
- permeability is almost never complex, except in the case of lossy ferromagnetic materials such as iron, cobalt, nickel, and their alloys
- relationships between period T, frequency f (or ω), wavelength λ, and phase constant k (also called the wave number) of waves
- **E** and **H** are *in* phase if medium is lossless
- speed of wave found from $\dfrac{\partial(\omega t - kz + \phi)}{\partial t} = 0$ or $\dfrac{\partial(\omega t - \beta z + \phi)}{\partial t}$; $u_p = \dfrac{\partial z}{\partial t}$, ϕ is const.

Polarization

- linear pol.
    - tip of E-field vector traces out a line segment over a full sinusoidal cycle (period)
    - both vector components (e.g., the x- and y-components of a wave traveling in z-direction) of E-field (and H-field) are in phase or 180° out of phase but might have unequal magnitudes (or one component could be zero)
    - two mutually perpendicular linear polarizations are possible. Implication: an antenna oriented to align with one polarization will be insensitive to the other.
- circular pol. (CP)
    - tip of E-field vector traces out a circle over a full cycle (period)
    - both vector components of E-field (or H-field) have equal magnitude but are ±90° out of phase
    - left-hand and right-hand circular polarizations are possible; antennas that have LHCP are not compatible with antennas that have RHCP, and vice versa
    - H-field vector rotates at same rate and in same direction as E-field vector (angular velocity is $\omega = 2\pi f$) and is always perpendicular to E-field vector
- elliptical pol. (EP)
    - tip of E-field vector traces out an ellipse over a full cycle (period)
    - both vector components of E-field (or H-field) are out of phase by an amount other than 0°, 180°, or ±90° (or out of phase by ±90° with unequal magnitudes)
    - left-hand and right-hand elliptical polarizations are possible; LHEP might or might not be compatible with RHEP, and vice versa
    - H-field vector rotates at same rate and in same direction as E-field vector and is always perpendicular to E-field vector
- Linearly polarized antennas can receive some of the power contained in a CP or EP wave.
- CP antennas can receive some of the power contained in a linearly polarized or EP wave.
- It is rare for antennas and other signal sources to be intentionally designed to produce elliptical polarization, but it is highly likely that a linearly polarized or CP wave will become elliptically polarized to some extent after reflection, refraction, diffraction, and other interactions with objects in a real environment.
- Most antennas are linearly polarized, but there are important applications for CP antennas:
    - broadcasting (so that viewer/listeners can use linearly polarized antennas of any orientation)
    - VHF/UHF/low microwave satellite communications (to compensate for Faraday rotation)
    - data multiplexing (parallel channels using opposite-sense CP antennas do not interfere with each other)
    - helix antennas are relatively easy to construct and look cool!

<u>Relevant course material:</u>

HW: #7 and #8
Reading: Assignments from Mar. 28 through Apr. 18, including
“The Small Loop Antenna”
“Derivation of the Antenna Effective Aperture Formula”
“Proof of Mutual Orthogonality of the Electric and Magnetic Fields and Direction of Propagation for Plane Waves”

A supplemental sheet with Tables 2-1, 2-2, 2-4, and 7-1 from the textbook (Ulaby and Ravaioli, 7^{th} ed.) plus several fundamental formulas will be made available to you during the exam. In addition, **four** 8.5 × 11-inch two-sided handwritten help sheets may be used during the exam.

This exam will focus primarily on the course outcomes listed below and related topics:

3. Relate the power density of a radiated electromagnetic wave to an antenna’s gain, radiation pattern, and applied input power. [focus on small loop antennas, and effective aperture]
4. Perform link budget calculations using the Friis transmission formula.
5. Mathematically express and/or analyze the polarization of an electromagnetic wave.

The course outcomes are listed on the Course Policies and Information sheet, which was distributed at the beginning of the semester and is available on the Syllabus and Policies page at the course web site. The outcomes are also listed on the Course Description page. Note, however, that some topics not directly related to the course outcomes could be covered on the exam as well.