ECEG 390 Theory and Applications of Electromagnetics Spring 2026

Policies and Review Topics for Exam #2

The following policies will be in effect for the exam. They will be included in a list of instructions and policies on the first page of the exam:

1. You will be allowed to use a non-wireless enabled calculator, such as a TI-89.
2. You will be allowed to use **two** 8.5 × 11-inch two-sided handwritten help sheets. No photocopied material or copied and pasted text or images are allowed. If there is a table or image from the textbook or some other source that you feel would be helpful during the exam and that is not included on the table and formula sheet that I will provide, please notify me.
3. All help sheets will be collected at the end of the exam but will be returned to you either immediately or soon after the exam.
4. Use of a help sheet that is not completely handwritten will result in an automatic 5-point score reduction. Help sheets that are handwritten on a tablet and then printed are acceptable.
5. If you begin the exam after the start time, you must complete it in the remaining allotted time. However, you may not take the exam if you arrive after the first student has completed it and left the room. The latter case is equivalent to missing the exam.
6. **You may not leave the exam room without prior permission except in an emergency or for an urgent medical condition. Please use the restroom before the exam.**

The exam will begin at 3:00 pm on Thursday, March 5 in Breakiron 165. You will have until 4:50 pm to complete the exam.

The following is a list of topics that could appear in one form or another on the exam. Not all of these topics will be covered, and it is possible that an exam problem could cover a detail not specifically listed here. However, this list has been made as comprehensive as possible. **You should be familiar with the topics on the review sheet for the previous exam as well.**

Although significant effort has been made to ensure that there are no errors in this review sheet, some might nevertheless appear. The textbook is the final authority in all factual matters, unless errors have been specifically identified there. You are ultimately responsible for obtaining accurate information when preparing for the exam.

Voltage/current maxima/minima on loaded transmission lines

- expressions for $\left|\tilde{V}(z)\right|$ and $\left|\tilde{I}(z)\right|$ and their derivations

$$\left|\tilde{V}(z)\right| = \left|V_0^+\right|\left[1+\left|\Gamma\right|^2 + 2\left|\Gamma\right|\cos\left(\theta_r + 2\beta z\right)\right]^{1/2}$$

$$\left|\tilde{I}(z)\right| = \frac{\left|V_0^+\right|}{Z_0}\left[1+\left|\Gamma\right|^2 - 2\left|\Gamma\right|\cos\left(\theta_r + 2\beta z\right)\right]^{1/2}$$

- voltage maxima (current minima) occur where $\cos(\theta_r + 2\beta z) = +1$, so (using $\beta = 2\pi/\lambda$)

$$\theta_r = -\frac{4\pi}{\lambda} z_{\max} \pm 2\pi n \quad n = 0, 1, 2, \ldots \quad \text{and} \quad z_{\max} = -\frac{\theta_r \lambda}{4\pi} \pm n\frac{\lambda}{2} \quad n = 0, 1, 2, \ldots$$

- voltage minima (current maxima) occur where $\cos(\theta_r + 2\beta z) = -1$, so

$$\theta_r = -\frac{4\pi}{\lambda} z_{\min} - \pi \pm 2\pi n \quad n = 0, 1, 2, \ldots \quad \text{and} \quad z_{\max} = -\frac{\theta_r \lambda}{4\pi} - \frac{\lambda}{4} \pm n\frac{\lambda}{2} \quad n = 0, 1, 2, \ldots$$

- voltage maxima and minima separated by $\lambda/4$
- current maxima and minima separated by $\lambda/4$
- voltage and current maxima and minima repeat every $\lambda/2$
- voltage maxima and current maxima separated by $\lambda/4$
- For the coordinate system that we are using, physically meaningful values of z are negative, and physically meaningful values of d (or l) are positive.
- VSWR, $d_{\max}$, and/or $d_{\min}$ on a mismatched line ($Z_L \neq Z_0$) can be used to find Z_L (basis of operation of slotted line)

Transmission line impedance calculations

- input impedance of lossless line with load:
  $$Z_{in}(-l) = Z_0 \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)}$$
- sometimes useful alternate form: $Z_{in}(-l) = \dfrac{\tilde{V}(-l)}{\tilde{I}(-l)} = Z_0 \dfrac{1+\Gamma e^{-j2\beta l}}{1-\Gamma e^{-j2\beta l}}$
- open-circuit load ($Z_L \to \pm\infty$): $Z_{in}(-l) = -jZ_0 \cot(\beta l)$

  if $l \ll \lambda$, then $Z_{in}(-l) = \dfrac{-jZ_0}{\tan(\beta l)} \approx \dfrac{-jZ_0}{\beta l} = -j\sqrt{\dfrac{L'}{C'}} \dfrac{1}{\left(\omega\sqrt{L'C'}\right)l} = \dfrac{1}{j\omega C' l}$
- short-circuit load ($Z_L = 0$): $Z_{in}(-l) = jZ_0 \tan(\beta l)$

  if $l \ll \lambda$, then $Z_{in}(-l) \approx jZ_0 \beta l = j\sqrt{\dfrac{L'}{C'}}\left(\omega\sqrt{L'C'}\right)l = j\omega L' l$
- matched line ($Z_L = Z_0$): $Z_{in}(-l) = Z_0$
- quarter-wave matching section ($l = \lambda/4$): $Z_{in}(-l) = \dfrac{Z_0^2}{Z_L}$
- half-wave section ($l = n\lambda/2$): impedances repeat every $\lambda/2$ along line
- purely reactive loads ($Z_L = jX$): $|\Gamma| = 1$; Z_{in} is purely imaginary
- "electrically short" lines (i.e., $l \ll \lambda$): $Z_{in} \approx Z_L$; this is the "circuit theory" limit; valid only if $Z_0 \gg \left|Z_L \tan(\beta l)\right|$ and $\left|Z_L\right| \gg \left|Z_0 \tan(\beta l)\right|$ (i.e., denominator of formula for Z_{in} above is dominated by Z_0 and numerator is dominated by Z_L)
- differences between Z_0 and Z_{in} and Z_L
- VSWR along line (especially along matching sections) changes at transitions, such as changes in Z_0 or locations where a device or component (such as an R, L, C, or antenna) is connected in parallel or in series with the line
- lossy lines: $Z_{in}(-l) = Z_0 \dfrac{Z_L + Z_0 \tanh(\gamma l)}{Z_0 + Z_L \tanh(\gamma l)}$, where $\gamma = \alpha + j\beta$ and Z_0 is complex
- calculation of V_0^+ and/or $\tilde{V}_{in}$, and the meanings of the two quantities

  textbook formula: $V_0^+ = \tilde{V}_g \left(\dfrac{Z_{in}}{Z_g + Z_{in}}\right)\left(\dfrac{1}{e^{j\beta l} + \Gamma e^{-j\beta l}}\right)$ and $\tilde{V}_{in} = \tilde{V}_g \left(\dfrac{Z_{in}}{Z_g + Z_{in}}\right)$

  alternate formula: $V_0^+ = \tilde{V}_g \dfrac{Z_0}{\left(Z_0 + Z_g\right)e^{j\beta l} + \left(Z_0 - Z_g\right)\Gamma e^{-j\beta l}}$ (useful when $Z_{in} = 0$)

  Note: if $Z_g = Z_0$, then $V_0^+ = \dfrac{1}{2}\tilde{V}_g e^{-j\beta l}$

- electrical length of line (in degrees, radians, or wavelengths) vs. physical length (in cm, m, etc.)
- determination of total phasor or time-domain voltage or current at specific locations along line or their components [i.e., $\tilde{V}_{fwd}(z)$ and $\tilde{V}_{ref}(z)$ or $\tilde{I}_{fwd}(z)$ and $\tilde{I}_{ref}(z)$]; use

  $\tilde{V}_{tot}(z) = V_0^+\left(e^{-j\beta z} + \Gamma e^{j\beta z}\right)$ and $\tilde{I}_{tot}(z) = \dfrac{V_0^+}{Z_0}\left(e^{-j\beta z} - \Gamma e^{j\beta z}\right)$
- cascaded line sections (one follows another) of different characteristic impedances
- equivalent impedance at junction of two or more line sections (i.e., "shunt" connections)
- input impedance when loads/devices are placed at different locations along line (not just at the end)

Impedance matching

- typical goals of impedance matching:
  - maximum power transfer
  - specific impedance at input end of line required for proper operation of signal source
  - to prevent reflections
  - combination of two or more of the above
- quarter-wave matching sections (characteristic impedance = Z_{0Q})
  - if Z_L is purely real and target matching impedance Z_{in} is purely real (latter is usually the case), use a quarter-wave section with $Z_{0Q} = \sqrt{Z_{in} Z_L}$
  - for complex loads:
    - can connect stub or lumped element at load to cancel any load reactance (susceptance)
    - alternatively, can find location of voltage max (at $z = -d_{max}$) or min (at $z = -d_{min}$), where Z_{in} is purely real (n = integer):

      voltage max: $\theta_r - 2\beta d_{max} = 2\pi n$ and $Z(-d_{max}) = Z_0 \dfrac{1+|\Gamma|}{1-|\Gamma|}$

      voltage min: $\theta_r - 2\beta d_{min} = (2n+1)\pi$ and $Z(-d_{min}) = Z_0 \dfrac{1-|\Gamma|}{1+|\Gamma|}$

      if matching to main line of characteristic impedance Z_0, use

      $$Z_{0Q} = \sqrt{Z(-d_{max}) Z_0} \quad \text{or} \quad Z_{0Q} = \sqrt{Z(-d_{min}) Z_0}$$
- definitions of admittance, conductance, and susceptance: $Y = G + jB$, where $Y = 1/Z$
- $B = -1/X$ for a pure reactance or susceptance
- $G = 1/R$ for a pure resistance or conductance
- wave admittance: $Y_{in}(-l) = Y_0 \dfrac{Y_L + jY_0 \tan(\beta l)}{Y_0 + jY_L \tan(\beta l)}$ and $Y_{in}(-l) = Y_0 \dfrac{1 - \Gamma e^{-j2\beta l}}{1 + \Gamma e^{-j2\beta l}}$
- series-element matching
  - an "element" is typically a capacitor, inductor, or stub (usually a C or L)
  - place element at a location where $\text{Re}\{Z_{in}\} = Z_0$
  - two possible locations every $\lambda/2$: $l_{main}^{series} = \dfrac{\lambda}{4\pi}\left[\theta_r \pm \cos^{-1}\left(|\Gamma|\right)\right]$, where $\Gamma = |\Gamma| e^{j\theta_r}$
  - can also use any location $n\lambda/2$ away from those given by formula (n = integer)

  - input reactances at those points are given by

    $$X_{in}^{series} = \mathrm{Im}\left\{Z_{in}\left(-l_{main}^{series}\right)\right\} = \mp\frac{2|\Gamma|}{\sqrt{1-|\Gamma|^2}}Z_0$$

    "+" solution to l_{main} formula corresponds to "–" solution to X_{in} formula
  - set $X_{element} = -X_{in}$
- shunt-element matching
  - an "element" is typically a capacitor, inductor, or stub (usually a stub)
  - place element at a location where $\mathrm{Re}\{Y_{in}\} = Y_0 = 1/Z_0$
  - two possible locations every $\lambda/2$: $l_{main}^{shunt} = \frac{\lambda}{4\pi}\left[\theta_r \pm \cos^{-1}\left(-|\Gamma|\right)\right]$, where $\Gamma = |\Gamma|e^{j\theta_r}$
  - can also use any location $n\lambda/2$ away from those given by formula (n = integer)
  - input susceptances at those points are given by

    $$B_{in}^{shunt} = \mathrm{Im}\left\{Y_{in}\left(-l_{main}^{shunt}\right)\right\} = \pm\frac{2|\Gamma|}{\sqrt{1-|\Gamma|^2}}Y_0$$

    "+" solution to l_{main} formula corresponds to "+" solution to B_{in} formula
  - inductive susceptance is negative; capacitive susceptance is positive
  - set $B_{stub} = -B_{in}$
- short- or open-circuited stubs cancel input susceptance B_{in} or input reactance X_{in}
  - goal is to make $B_{stub} = -B_{in}$ or X_{stub} (or $X_{element}$) $= -X_{in}$
  - short-circuited stub: $l_{stub,sc} = \frac{\lambda}{2\pi}\tan^{-1}\left(\frac{X_{stub,sc}}{Z_0}\right) = \frac{\lambda}{2\pi}\tan^{-1}\left(\frac{-Y_0}{B_{stub,sc}}\right)$
  - open-circuited stub: $l_{stub,oc} = \frac{\lambda}{2\pi}\tan^{-1}\left(\frac{B_{stub,oc}}{Y_0}\right) = \frac{\lambda}{2\pi}\tan^{-1}\left(\frac{-Z_0}{X_{stub,oc}}\right)$
- add $\lambda/2$ if formula for l_{main} or l_{stub} gives a negative length (arises from the 360° ambiguity with the $\tan^{-1}$ and $\cos^{-1}$ functions)
- use care when evaluating $\tan^{-1}$; check quadrant!
- can use lumped-element (C and/or L only) matching networks instead of stubs in many cases (usually at low microwave frequencies and below)
- Z_0 of stub does not have to equal Z_0 of main line, but it usually does
- be able to find VSWR along stub, along line between matching element and load, and along line between signal source and matching element (requires the calculation of Γ at various points)

Power flow along transmission lines

- be able to calculate power delivered by voltage/current source (generator), absorbed in source resistance (or impedance), delivered to input of line, and/or absorbed by load
- be able to calculate incident power P_{inc} and reflected power P_{ref} (sometimes represented as P^i_{av} and P^r_{av} because the formulas give values for time-average power)
- available power (P_A) = maximum power that can be delivered by source; this occurs when the load on the source (which might not equal the load on a transmission line) equals the complex conjugate of the Thévenin equivalent source impedance (Z_g):

  $$P_A = \frac{|\tilde{V}_g|^2}{8R_g},$$ where $\tilde{V}_g$ = phasor source (generator) voltage in peak units; $R_g = \mathrm{Re}\{Z_g\}$
- $P_{inc} = P_A$ if $Z_0 = Z_g$ (char. impedance = generator impedance)

- incident (P_{inc}) vs. reflected (P_{ref}) power and their relationship to power delivered to load

  $$P_L = P_{inc} - P_{ref} = \frac{\left|V_0^+\right|^2}{2Z_0}\left(1-\left|\Gamma\right|^2\right)$$

  Textbook uses $P_L = P_{inc} + P_{ref}$, where P_{ref} is negative since it is "delivered" by the load to the transmission line. Either interpretation ($\pm P_{ref}$) is fine; pay attention to context.
- Acceptable to represent P_{ref} as a positive quantity if $P_L = P_{inc} - P_{ref}$ is used
- conservation of power applies everywhere at all times
- calculation of time-average real power from phasor quantities:
  [Textbook uses P_{av} to represent P_L (power delivered to load), whereas P_{av} is used here to denote time-average power in the general sense.]

  $$P_{av} = \frac{1}{2}\mathrm{Re}\left\{\tilde{V}\tilde{I}^*\right\} = \frac{1}{2}\left|\tilde{I}\right|^2 R = \frac{1}{2}\left|\tilde{V}\right|^2 \mathrm{Re}\left\{\frac{1}{Z^*}\right\} = \frac{1}{2}\left|\tilde{V}\right|^2 \mathrm{Re}\left\{Y^*\right\} = \frac{1}{2}\left|\tilde{V}\right|^2 G$$

  where voltage and current have peak units, not rms units (omit the "2" for the rms case)

  Note: $\mathrm{Re}\left\{\frac{1}{Z^*}\right\} = \mathrm{Re}\left\{\frac{1}{(R+jX)^*}\right\} = \mathrm{Re}\left\{\frac{1}{R-jX}\right\} \neq \frac{1}{R}$, if Z is complex (watch out!)
- net average real power delivered to load: $P_L = P_{inc}\left(1-\left|\Gamma\right|^2\right)$, where $P_{inc} = \dfrac{\left|V_0^+\right|^2}{2Z_0}$.
- return loss (in dB) = $-10 \log |\Gamma|^2 = -20 \log |\Gamma|$
  Note that return loss is a *positive* value in dB because $|\Gamma| < 1$. The corresponding reflection coefficient expressed in dB has a *negative* value, again because $|\Gamma| < 1$. The definition of return loss given here is an industry standard; however, some authors will carelessly express return loss as a negative value because they calculate it as $20 \log |\Gamma|$ (i.e., without the leading minus sign).
- return loss ≈ 10 dB, if $|\Gamma| = 0.333$ (VSWR = 2). For this case, $P_{ref} \approx 10\%$ of P_{inc}. It is a widely accepted threshold for a "good" impedance match.
- reactive power associated with transmission lines
  - represents energy stored and released during each half of the AC cycle by the distributed capacitance and inductance along the line
  - can calculate using

    $$Q_{line} = Q_{in} - Q_L = \frac{\left|\Gamma\right|\left|V_0^+\right|^2}{Z_0}\left[\sin\left(\theta_r - 2\beta l\right) - \sin\theta_r\right]$$

    where Q_{in} = input reactive power and Q_L = load reactive power
  - $Q_{line} = 0$ if $|\Gamma| = 0$ or $2\beta l = 2\pi n$, where n is any integer; the latter condition corresponds to $l = n\lambda/2$
  - if $Q_{line} = 0$, the reactive powers associated with the distributed capacitance and inductance along the line are still non-zero, but they exactly cancel
  - $Q_{in} = 0$ if the input impedance of the line is purely real
  - $Q_L = 0$ if the load impedance is purely real

<u>Relevant course material:</u>

HW: #3 and #4
Reading: Assignments from Feb. 5 through Feb. 26, including the supplemental readings
“Equivalent Inductance and Capacitance of Short Transmission Line Stubs”
“Alternative Formula for Calculating the Forward Voltage Coefficient”
“Impedance Matching Using Series and Parallel Transmission Line Elements”
“Reactive Power Calculations for Loaded Transmission Lines”

This exam will focus primarily on the course outcomes listed below and related topics:

1. Predict voltages, currents, and/or power flow along a transmission line given the line parameters and the signal source and load connected to the line.
2. Design a transmission line-based impedance matching system.

The course outcomes are listed on the Course Policies and Information sheet, which was distributed at the beginning of the semester and is available on the Syllabus and Policies page at the course web site. The outcomes are also listed on the Course Description page. Note, however, that some topics not directly related to the course outcomes could be covered on the exam as well.