Homework Assignment #7 – due via Moodle at 11:59 pm on Friday, Nov. 21, 2025 [Graded Prob. 2 revised 11/20/25]

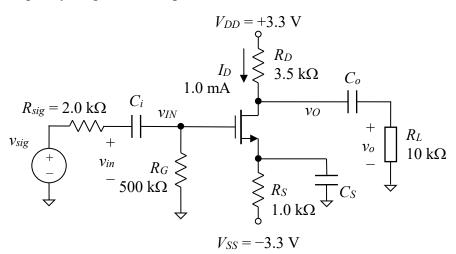
Instructions, notes, and hints:

You may make reasonable assumptions and approximations to compensate for missing information, if any. Provide the details of all solutions, including important intermediate steps. You will not receive credit if you do not show your work.

The first few problems will be graded and the rest will not be graded. Only the graded problems must be submitted by the deadline above. Do not submit the ungraded problems.

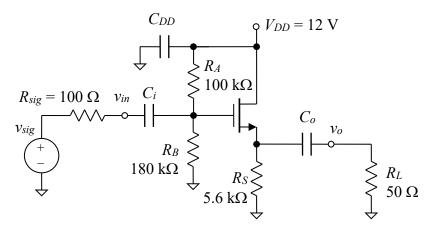
Graded Problems:

1. The *n*-channel MOSFET in the circuit shown below has $k_n = 800 \,\mu\text{A/V}^2$, $V_t = 0.70 \,\text{V}$, and $V_A = 40 \,\text{V}$ ($\lambda = 0.025 \,\text{V}^{-1}$). The circuit uses a bipolar (±3.3 V) power supply. The resistor values have been chosen to produce a quiescent drain current I_D of 1.0 mA and a quiescent value for V_{DS} that is 0.50 V above the triode-saturation boundary (defined by $V_{DS} = V_{GS} - V_t$). Channel-length modulation was ignored for the bias analysis. Find the small-signal voltage gain v_o/v_{in} of the amplifier with the effect of the small-signal output resistance r_o included. Use a small-signal model for your analysis. All three capacitors have negligible reactances over the operating frequency range of the amplifier.

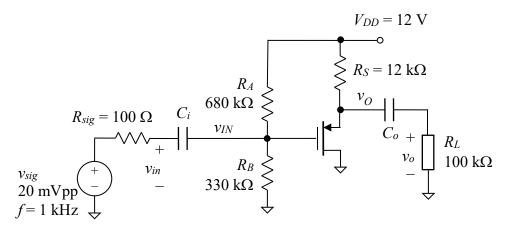


2. [boldface text revised 11/20/25] For any type of MOSFET amplifier circuit, the small-signal constraint is $|v_{gs}(t)| \ll 2V_{OV}$. That is, the magnitude of the signal component of v_{GS} at any point in time must satisfy the constraint. For the source follower circuit considered in the next problem, find the corresponding constraint on the small-signal input voltage v_{in} . In other words, find the maximum value of $|v_{in}|$ so that v_{gs} does not violate the small-signal condition. Note that you must find a relationship between v_{in} and v_{gs} and then use the constraint on v_{gs} as a step toward finding one for v_{in} .

3. Using small-signal analysis, find the small-signal voltage gain v_o/v_{in} of the source follower depicted below with the effect of the small-signal output resistance r_o included. The MOSFET has the parameter values $k_n = 200 \text{ mA/V}^2$, $V_t = 2.0 \text{ V}$, and $V_A = 50 \text{ V}$ ($\lambda = 0.020 \text{ V}^{-1}$). All three capacitors have negligible reactances at the operating frequency.



4. For the PMOS-based source follower circuit depicted below, find the numerical value of the quiescent drain current I_D . Also find the small-signal transconductance g_m . The MOSFET's parameters are $k_p = 100 \text{ mA/V}^2$ and $V_{tp} = -2.0 \text{ V}$. You may assume that $\lambda = 0$ (i.e., that channel-length modulation is negligible). *Note*: The formula for I_D that applies for NMOS devices with source degeneration will not work for this problem. You will have to modify the formula to work for PMOS devices or come up with another way to find I_D .

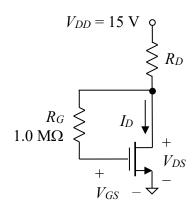


5. For the source follower considered in the previous problem, use an appropriate small-signal circuit model to find a symbolic expression for the voltage gain $A_v = v_o/v_{in}$, and then find the numerical value of the voltage gain with and without the MOSFET output resistance r_o included. The capacitors have negligible reactances at the signal frequency. The MOSFET has the parameters $k_p = 100 \text{ mA/V}^2$, $V_{tp} = -2 \text{ V}$, and $V_A = 40 \text{ V}$ ($\lambda = 0.025 \text{ V}^{-1}$).

Ungraded Problems:

The following problems will not be graded, but you should attempt to solve them on your own and then check the solutions. Do not give up too quickly if you struggle with one or more of them. Move on to a different problem and then come back to the difficult one after a few hours.

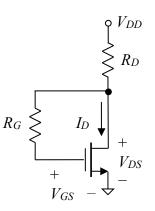
1. The circuit shown at right is an alternative MOSFET bias network that uses a feedback resistor R_G between the gate and the drain terminals. It is used in situations when the source terminal must have a direct connection to ground. An analysis of the circuit to determine an expression for the quiescent drain current I_D is the focus of the next ungraded problem. The value of R_G is not usually critical in this type of circuit, but there are good reasons to make it a large value, so assume that $R_G = 1.0 \text{ M}\Omega$. If $k_n = 200 \text{ mA/V}^2$ and $V_t = 2.0 \text{ V}$ for the MOSFET, find the value of R_D required to set the quiescent drain current to 500 μ A. Also find the resulting quiescent value of V_{DS} .



2. The circuit shown below is an alternative MOSFET bias network that uses a feedback resistor R_G between the gate and the drain terminals. This technique can be used when stable bias is required but the source terminal must be grounded, as is sometimes the case in high-frequency circuits for which a source degeneration resistor in parallel with a bypass capacitor might not be able to provide a sufficiently low-impedance path between the source and ground. That is, the parallel RC combination might have too much stray impedance at high frequencies. Show that the quiescent drain current is given by the expression for I_D below. Hint: Start the derivation with the KVL expression and its modifications shown above the expression for I_D . Note that $V_{DS} = V_{GS}$ (bias values) because zero DC current flows through R_G .

$$\begin{split} V_{DD} &= I_D R_D + V_{DS} = I_D R_D + V_{GS} \\ V_{DD} &= \frac{1}{2} k_n R_D \left(V_{GS} - V_t \right)^2 + V_{GS} \\ V_{DD} - V_t &= \frac{1}{2} k_n R_D \left(V_{GS} - V_t \right)^2 + V_{GS} - V_t \\ V_{DD} - V_t &= \frac{1}{2} k_n R_D V_{OV}^2 + V_{OV} \end{split}$$

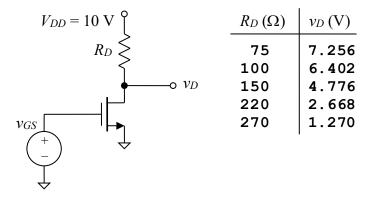
$$I_{D} = \frac{1 + k_{n} R_{D} \left(V_{DD} - V_{t}\right) - \sqrt{1 + 2k_{n} R_{D} \left(V_{DD} - V_{t}\right)}}{k_{n} R_{D}^{2}}$$



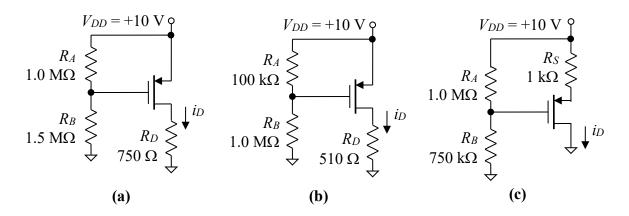
3. The test circuit shown below is used to measure the parameters k_n , V_t , and λ of an n-channel MOSFET. The data shown next to the figure were obtained as the drain resistor R_D was varied in value while v_{GS} was held at a value of 2.900 V using a precision power supply. Find the value of λ from the given data. You do not have to find the values of k_n and V_t , although you may do so if you wish. Hint #1: Equation (5.23) of the textbook (Sedra & Smith, 8^{th} ed.) appears to be applicable here, but it is an approximation. A more accurate form is

$$i_D = \frac{1}{2} k_n v_{OV}^2 [1 + \lambda (v_{DS} - v_{OV})],$$

where $v_{OV} = v_{GS} - V_t$ (v_{OV} is the "overvoltage"). The channel-length modulation parameter λ is equal to the reciprocal of the Early voltage V_A . Figure 5.17 in the textbook should help you see the significance of this fact as it applies to solving this problem.

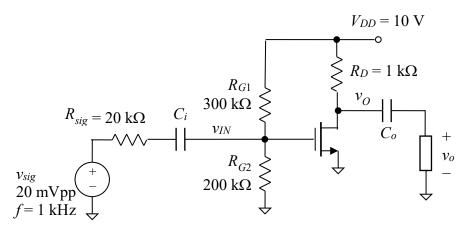


4. Find the regions of operation of the PMOS devices in the circuits shown below, and find the drain current i_D and source-to-drain voltage v_{SD} for each case. The MOSFETs' parameters are $k_p = 10 \text{ mA/V}^2$ and $V_{tp} = -2.0 \text{ V}$. You may assume that $\lambda = 0$ (i.e., there is no channel-length modulation).



- 5. The common-source amplifier circuit shown below is not practical because there is no source degeneration resistor to stabilize the bias voltages and currents. However, some insights into amplifier operation can be gained from it. A sinusoidal signal $v_{sig}(t)$ with the indicated properties is applied to the input. (The phase doesn't matter, so assume that it is zero.) The MOSFET has the parameter values $k_n = 1.0 \text{ mA/V}^2$ and $V_t = 1.0 \text{ V}$. Since this is an amplifier, you may assume that the designers intended for the MOSFET to operate in the saturation region at all times. You may also assume that $\lambda = 0$, that all of the capacitors have negligible reactances at the signal frequency, and that the load acts like an open circuit.
 - **a.** Use an appropriate small-signal model of the circuit to find the numerical value of the small-signal voltage gain $A_v = v_o/v_{in}$, assuming linear operation around the quiescent point.
 - **b.** Suppose that the values of resistors R_{G1} and R_{G2} are changed so that the quiescent gate-to-source voltage V_{GS} is just barely above the threshold voltage V_t (i.e., the MOSFET is on the edge of the cut-off region). Find the value of the small-signal voltage gain A_v at the new operating point.
 - c. Now suppose that the quiescent gate-to-source voltage V_{GS} has the value that causes the MOSFET to operate at the boundary between the saturation and triode regions, that is, at point B in Fig. 7.2b of Sedra & Smith, 7^{th} ed. Find the small-signal voltage gain value for this case.

Note that in practice the amplifier should not be operated in either of the conditions represented by parts b and c because the output voltage has no room to swing above and below its quiescent value without the MOSFET entering the cut-off (part b) or triode (part c) region. The voltage gains found in the two parts represent theoretical lower and upper limits for this particular circuit.



6. Shown on the next page is a common-source (CS) amplifier that uses four-resistor biasing. The load has an equivalent resistance of 2.5 M Ω , so it can be ignored for gain calculations. The circuit uses the biasing rule-of-thumb $I_DR_D = I_DR_S = V_{DS} = V_{DD}/3$. The quiescent drain current is around 1.0 mA, and the voltage gain is $A_v = v_o/v_{in} = -40 \text{ V/V}$. The magnitude of the input voltage v_{in} must be less than 40 mV to satisfy the small-signal condition (a factor of 1/10 corresponds to "much less than").

Higher gain can be achieved by distributing the quiescent voltages differently. Since $R_L \gg R_D$, then

$$A_{v} \approx -g_{m}R_{D} = -\sqrt{2k_{n}I_{D}}R_{D} = -\sqrt{2k_{n}I_{D}}\left(\frac{I_{D}R_{D}}{I_{D}}\right) = -\sqrt{\frac{2k_{n}}{I_{D}}}\left(I_{D}R_{D}\right),$$

which suggests that the gain of a CS amplifier can be increased by increasing the quiescent voltage across R_D (i.e., the voltage I_DR_D) and/or decreasing the quiescent drain current. However, increasing the voltage I_DR_D can push the quiescent point closer to the triode region unless the voltage across R_S (the voltage I_DR_S) is reduced as well because, by KVL,

$$V_{DD} = I_D R_D + V_{DS} + I_D R_S \quad \rightarrow \quad V_{DS} = V_{DD} - I_D R_D - I_D R_S \,, \label{eq:VDD}$$

but we need to maintain $V_{DS} > V_{GS} - V_t$.

The *total* voltage v_{DS} fluctuates if a signal is present, so the *bias* value V_{DS} must be comfortably above V_{OV} . Reducing the voltage across R_S results in less stable bias conditions, so this approach represents a trade-off. If the value of I_D is reduced to increase the gain, then the small signal condition becomes more constrained because

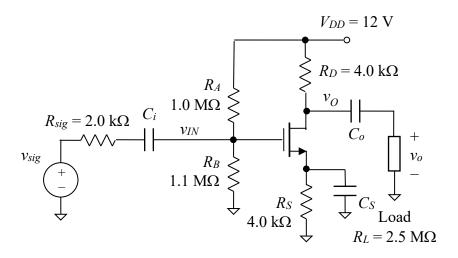
$$I_D = \frac{1}{2} k_n (V_{GS} - V_t)^2 = \frac{1}{2} k_n V_{OV}^2 \rightarrow V_{OV} = \sqrt{\frac{2I_D}{k_n}} \quad \text{and} \quad |v_{gs}| << 2(V_{GS} - V_t) = 2V_{OV}.$$

A smaller I_D leads to a smaller V_{OV} , which in turn leads to a tighter restriction on the magnitude of v_{gs} , which in the CS amplifier below is equal to the small-signal voltage v_{in} .

The designers of the amplifier decide to increase the gain by increasing the voltage across R_D to $0.4V_{DD}$ and decreasing the voltage across R_S to $0.26V_{DD}$ to compensate. They will also reduce I_D from 1.0 mA to 500 μ A. Find:

- **a.** the new values of R_A , R_B , R_D , and R_S to meet the new specifications with the constraint that $R_A||R_B \ge 500 \text{ k}\Omega$
- **b.** the new voltage gain $A_v = v_o/v_{in}$. Use a small-signal model for your analysis.
- c. the new limit on the input voltage magnitude $|v_{in}|$, assuming that a factor of 1/10 corresponds to "much less than"

The MOSFET's parameters are $k_n = 50 \text{ mA/V}^2$ and $V_t = 2.0 \text{ V}$. All of the capacitors have negligible reactances at the signal frequency.



Circuit diagram for Ungraded Prob. 6