Lab 13

Charged Particles in Fields

Continuing Objectives

1. Be able to identify sources of experimental uncertainty in a measurement.

2. Know how to determine experimental uncertainties (multiple measurements of the same quantity, propagation of errors, etc.).

3. Be able to write an experimental result (including correct number of significant digits, uncertainty, units).

4. Be able to make careful measurements to ensure reproducible results.

5. Know how to keep a clear and organized record, including an introduction (with purpose of lab and appropriate laws or equations), apparatus sketch, table of raw data and calculated quantities, and a good conclusion or summary.

7. Know how to make comparisons: are two measured quantities equal? Is a measured quantity statistically equivalent to a theoretical value?

10. Be able to work with physical vector quantities.

Introduction

The object of these experiments is to understand the effects of electric and magnetic fields on charges and to determine the charge-to-mass ratio, e/m, for electrons.

The first part of the experiment uses a cathode ray tube (CRT) to produce a beam of charged particles, which you will deflect with magnetic fields. CRTs form the screen of older computer monitors, television sets, and oscilloscopes. Some qualitative experiments will help you understand the relationship between the directions of the velocity of the charged particles, the magnetic field, and the deflecting force. This will enable you to determine the type of charged particle in the CRT beam and the direction of the Earth's magnetic field.

The second part of the experiment uses an apparatus which fires a beam of electrons, each at the same velocity, into a uniform magnetic field of known strength. These electrons will be deflected into a visible circular path, whose radius enables you to calculate the charge-to-mass ratio, e/m.

This lab gives you experience with many important concepts. Specifically, you will see how a charged particle accelerates in an electric field and how potential difference results in the change in kinetic energy of a charged particle. You'll also see how to apply the right hand rule to determine the direction of the magnetic force acting on a particle.

Part I: Relationship Between Velocity, Magnetic Field, and Deflecting Force

Theory

The magnetic force on a charge q moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ is

$$\vec{F} = q\,\vec{v} \times \vec{B}. \tag{13.1}$$

The direction of the force is determined by the right hand rule. The magnitude of this cross-product is

$$F = q\,v\,B\,\sin\theta, \tag{13.2}$$

where θ is the angle between $\vec{v}$ and $\vec{B}$. The direction of this cross-product you determine as described in Lab 12 for Eq. (12.1).

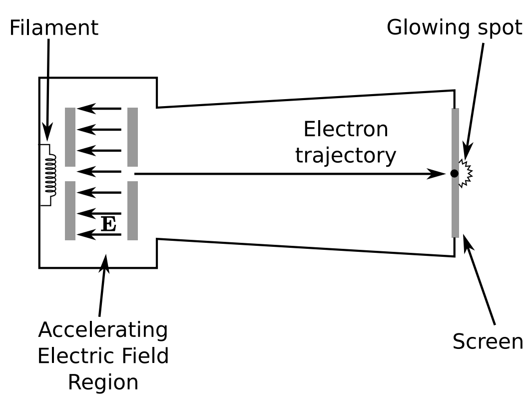


Figure 13.1: Diagram of cathode ray tube (CRT) with no magnetic field.

Procedure

Each lab bench is equipped with a cathode ray tube (CRT) and power supply. In the CRT, charged particles leave the surface of a hot filament at the back of the tube and immediately enter an electric field pointing along the axis of the tube and extending for a short portion of the tube. The electric field accelerates each charged particle towards the front of the tube. Once the particles leave the electric field, they travel in a beam toward the screen with constant velocity. The charged particles strike the inside of the glass at the front of the tube (the screen). This is coated with a fluorescent material, which glows briefly wherever an electron strikes it. With no external magnetic fields, the glowing spot produced by the beam would be located in the center of the screen. A permanent magnet, current-carrying coils, or the Earth's magnetic field can deflect it.

1. **If you have a plastic CRT on your desk:** Turn on the CRT. Focus the spot using the "BEAM FOCUS" knob on the CRT mount and note the spot's location.

2. **If you have a glass CRT on your desk:** Turn the main power switch on, and then turn the "Anode Power" switch on. You may ignore the "Polarity" and "Deflection" settings. Note the spot's location on the flat end of the tube. Please handle this apparatus carefully!

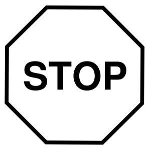
3. Now determine where the N and S poles of each permanent magnet are. Place one of the bar magnets between the middle and front of the CRT so that the magnet points perpendicular to the direction in which the charged particles are moving. Note which pole is closest to the beam and describe the deflection of the spot (a sketch of the screen could be useful). Invert the magnet and describe the deflection of the spot. Place two magnets on opposite sides of the beam and describe the deflection; try all combinations N-N, S-S, S-N, and N-S. Note that near the North pole of a bar magnet, the magnetic field points away from the magnet, and that near the South pole of a bar magnet, the magnetic field points toward the magnet.

4. Place a current-carrying coil between the middle and front of the CRT. Orient the coil (try the many possibilities) so that the spot on the screen is deflected. Is the effect of the coil similar to that of the permanent magnet? If so, where are the effective poles for the coil? How could you tell which is N and which is S?

5. From your observations, determine the sign of the charge of the particles in the CRT beam. Describe how you did this. Determine whether the electric field in the CRT points towards the back of the tube or towards the front of the tube.

6. Where is the spot on the CRT screen when you remove the magnetic fields caused by the bar magnets and the current carrying coils? Why isn't that spot

in the center of the screen? Move the CRT around until you find an orientation that causes the spot to be at the center of the tube (you can pick up or remove the CRT from its cradle to accomplish this task). Describe this orientation of the CRT. Determine the direction of the Earth's magnetic field (not just the horizontal component, but the whole field).



Discuss your results for the determination of the Earth's magnetic field with your instructor or TA.

Part II: Measuring the Charge-to-Mass Ratio of Electrons

Theory

The apparatus in this part of the lab shoots a beam of electrons with speed v into a magnetic field $\vec{B}$ so that initially they move perpendicular to the magnetic field. By considering the directions of the magnetic force and the motion, **explain** why these electrons will continue to *move in a circle with constant speed*.

From Eq. (13.2) we follow that an electron moving in a uniform magnetic field at right angles to the field is acted on by a force with magnitude

$$F = evB, \tag{13.3}$$

where e is the magnitude of the charge of the electron, v is the speed of the electron, and B is the magnitude of the magnetic field. We use Newton's second law $\vec{F}_{\text{net}} = m\vec{a}$ and also that for uniform circular motion the centripetal acceleration is $a = v^2/r$ to obtain

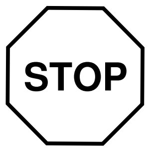
$$evB = m\frac{v^2}{r}, \tag{13.4}$$

where m is the mass of the electron and r is the radius of the circle.

The initial velocity of the electron results from the acceleration as it passes through a potential difference ΔV. For non-relativistic electrons initially at rest, energy conservation implies

$$\frac{1}{2}mv^2 = e\,\Delta V. \tag{13.5}$$

7. **Solve** Eq. (13.5) for v and **substitute** the result into Eq. (13.4) to find an expression for e/m in terms of B, r, and ΔV. Show this entire derivation in your lab notebook.



Discuss your derivation for $(e/m)_{expected}$ in terms of B, r, and ΔV with your TA or instructor.

8. Go to the back of the room to have a look at the apparatus, studying and noting each of its components. (More details will be explained in the Procedure section below.) Then return to your desk and read the description below to get an idea of what is going on when we use that apparatus.

The magnetic field for that apparatus will be produced by the current in the *Helmholtz coils*, a pair of thin, flat, circular coils of equal radius on a common axis with a separation of their planes equal to their radius. The strength of the field is

$$B = \frac{32\pi k_C}{c^2}\frac{NI}{\sqrt{125}\,a}, \tag{13.6}$$

where I is the current through the Helmholtz coils in amperes, N is the number of turns of wire in each coil, a is the mean radius of each coil, c is the speed of light, and k_C is Coulomb's force constant. (We do not need to derive this particular special case relationship for this lab.) The constants N and a are specific to the apparatus you will be using. For both table top models, $N = 130$. Combining Eq. (13.6) with your equation for e/m gives

$$\frac{e}{m} = \left(\frac{125}{512}\frac{c^4}{\pi^2 {k_C}^2}\frac{a^2}{N^2}\right)\frac{\Delta V}{I^2 r^2}. \tag{13.7}$$

Equation (13.7) is the working equation for this apparatus. **Make sure that you understand all the terms in this equation.** The quantity within parentheses is a constant for any given pair of Helmholtz coils. The value of r, the radius of the electron beam, can be varied by changing either the accelerating voltage, ΔV, or the Helmholtz-coil current, I.

9. To record your data, you will use an Excel sheet template found in the public netspace folder for Charged Particles in a Field. You should be able to access this from the desktop of your Lab PC. Copy this template into your netspace. Use this Excel sheet to do all your analysis for this lab. Keep constants at the beginning of this Excel sheet and refer to constants with absolute references (e.g. B3).

10. On this spreadsheet confirm that Eq. (13.7) reduces to

$$\frac{e}{m} = \left(2.47366 \times 10^{12}\,\frac{a^2}{N^2}\right)\frac{\Delta V}{I^2 r^2}. \tag{13.8}$$

Procedure

In each type of apparatus, electrons "boil" off the hot filament, which is heated by passing high current through it. The electrons leave the filament effectively at rest, and are accelerated through an adjustable potential difference, ΔV. The electrons are deflected by the magnetic field created by the Helmholtz coils. The field's strength can be adjusted by changing the current I in the Helmholtz coils. The apparatus is filled with mercury vapor; the electrons excite mercury atoms in collisions and the blue glow is the light given off as these excited atoms relax. Though the electrons themselves are not visible, the circular path of the electrons is therefore visible, and its radius, r, can be measured.

Of course, the Helmholtz coils aren't the only source of magnetic field. As you saw earlier in this lab, the Earth has a magnetic field. You can observe this in both set-ups, because even with no current in the Helmholtz coils and thus no magnetic field from the coils, the electron beam path is still a little curved.

1. Inspect your apparatus, identifying the filament and Helmholtz coils. Determine the direction of the magnetic field that the Helmholtz coils create. When you turn on the filament and the accelerating voltage, note the path of the electron beam. Convince yourself that the velocity of the charged particles is perpendicular to the magnetic field.

2. Detailed instructions are included with each apparatus; follow along and write into your lab notebook relevant information which you will need in your Excel spreadsheet. Information about the radius of the circular path, r, is given by the number which lights up on the crossbar. For example when the crossbar 8 lights up, it corresponds to a diameter of 8 cm, i.e. $r = 0.04$ m. In your lab notebook make a table with columns for the crossbar, r, ΔV, and I. The operating voltage range is $200 - 300\,\text{V}$ for the top models. For the open table model take measurements for $\Delta V = 200$ V for five different radii r, and then also take measurements for $\Delta V = 300$ V for five different radii. For the closed table model (in black box; Eisco) take measurements for five different r-values each for $\Delta V = 260$ V and for $\Delta V = 340$ V.

3. Before you return to your desk, make four measurements of a, the radius of the Helmholtz coils for your apparatus. Discuss your procedure with your partner and write down your method and results in your lab notebook. Talk to your lab instructor or TA if you are unsure of how to make this measurement.

Analysis

1. Put into your EXCEL sheet your measurements of a and crossbar, r, ΔV, and I in the provided cells (and table).
2. Determine the Helmholtz coil radius average a_{avg} and uncertainty Δa.
3. Determine the parentheses of Eq. (13.8) using a_{avg}.
4. Using Eq. (13.8) calculate e/m in the table (use a formula in the first entry and copy the formula for the whole column).
5. Calculate the mean, standard deviation (`STDEV` in Excel), and standard deviation of the mean of your results for e/m to determine an experimental value of e/m. What should you use for the uncertainty for this experimental value? (see $\Delta\left(e/m\right)_{\text{sm}}$)
6. When we determine e/m there is an additional uncertainty in e/m due to the uncertainty in the coil radius a. Use the uncertainty Δa and propagation of uncertainty to determine $\Delta(e/m)_a$, the uncertainty in e/m due to the uncertainty in a (see Appendix A on the PHYS 212 Lab Info page for a discussion of the propagation of uncertainties).
7. Discuss which error is larger, $\Delta(e/m)_a$ or the standard deviation of the mean $\Delta(e/m)_{\text{sm}}$ of Step 5.
8. Determine the combined error for (e/m), which is $\sqrt{(\Delta(e/m)_a)^2 + (\Delta(e/m)_{\text{sm}})^2}$
9. Report your best estimate for the value of e/m of an electron (with uncertainty).
10. Determine the expected value of e/m using the constants (pdf-file) on the PHYS 212 Lab Information web-page.
11. Ensure that both partners have a copy of this Excel sheet. Write into your lab notebook in which folder your EXCEL sheet is.
12. Print out a copy of your Excel spreadsheet and paste it into your notebook.
13. Compare your reported measured result with the expected value of e/m. Does your experimental result agree with the expected value?