**Physics 211** **Classical and Modern Physics** **Fall 2008**

## Spacetime Diagram Worksheet

### 1. The Fantastic Farmer Brown (Supp CH 3 #9)

Farmer Brown is carrying a ladder that he measures to be 20 lt-ns long and he is running through a barn whose rest length is 16 lt-ns. According to observers at rest with respect to the barn, Farmer Brown and his ladder are moving at $0.80c$. In this frame, then, the ladder is length contracted to

$$L_{\text{other}} = L_{\text{rest}}\sqrt{1 - v^2/c^2} = (20\,\text{lt-ns})\sqrt{1 - 0.80^2} = 12\,\text{lt-ns},$$

so the ladder fits in the barn. However, in Farmer Brown's frame the ladder is 20 lt-ns long and the *barn* is length contracted to

$$L_{\text{other}} = L_{\text{rest}}\sqrt{1 - v^2/c^2} = (16\,\text{lt-ns})\sqrt{1 - 0.80^2} = 9.6\,\text{lt-ns}.$$

Farmer Brown says the ladder does *not* fit in the barn. This seems like a paradox. Who is right?

We can resolve this paradox with a spacetime diagram. In the space to the right, sketch the world lines for the front of the barn, the back of the barn, the front of the ladder and the back of the ladder. *Hint:* make each grid spacing 4 ns or 4 lt-ns.

Then identify the events

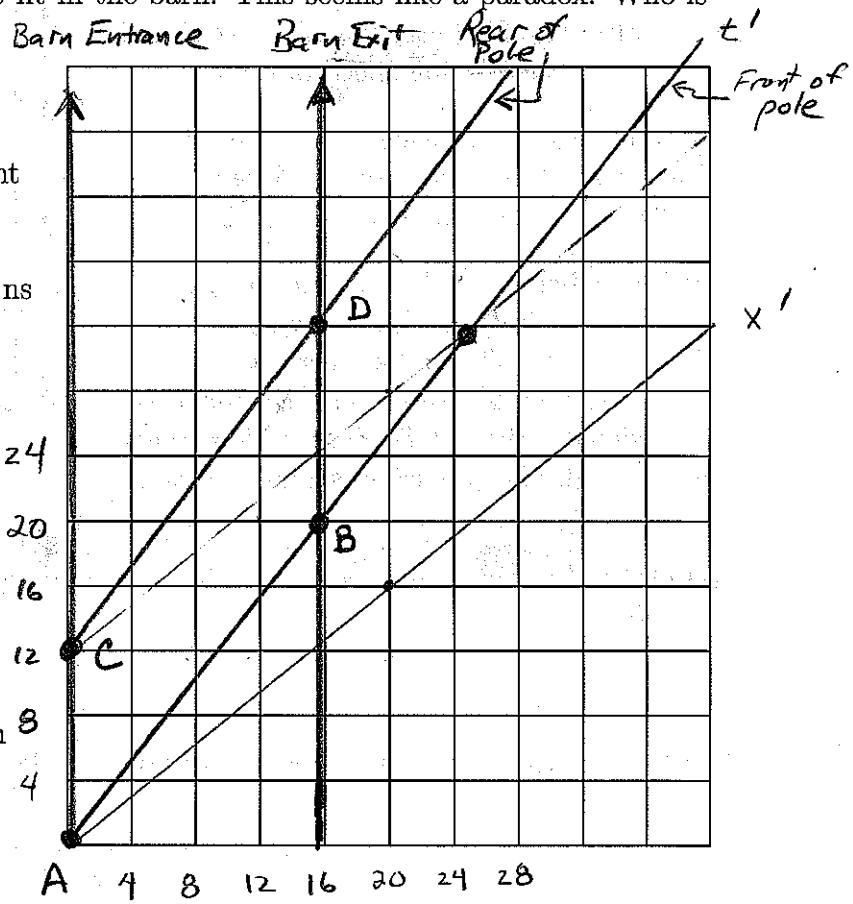
**A** – front of ladder enters barn,

**B** – front of ladder leaves barn,

**C** – back of ladder enters barn,

**D** – back of ladder leaves barn.

Now, use your diagram to evaluate the order of these events in the barn frame and in Farmer Brown's frame, and then explain how the event ordering resolves the paradox.




Reference Frame of Barn: A C B D

Rear of pole in barn before front exits. Consistent with pole length of 12 lt-ns

Reference Frame of Farmer: A B C D

Front of pole exits before rear enters.

**2. Relativistic Race**

Two rockets, the *Millenium Tortoise* and the *Millenium Hare*, are racing from Earth to Sirius, a distance of 8 lt-y away. Draw and label the worldlines for the Earth and Sirius, and then draw the worldlines described below carefully and to scale.

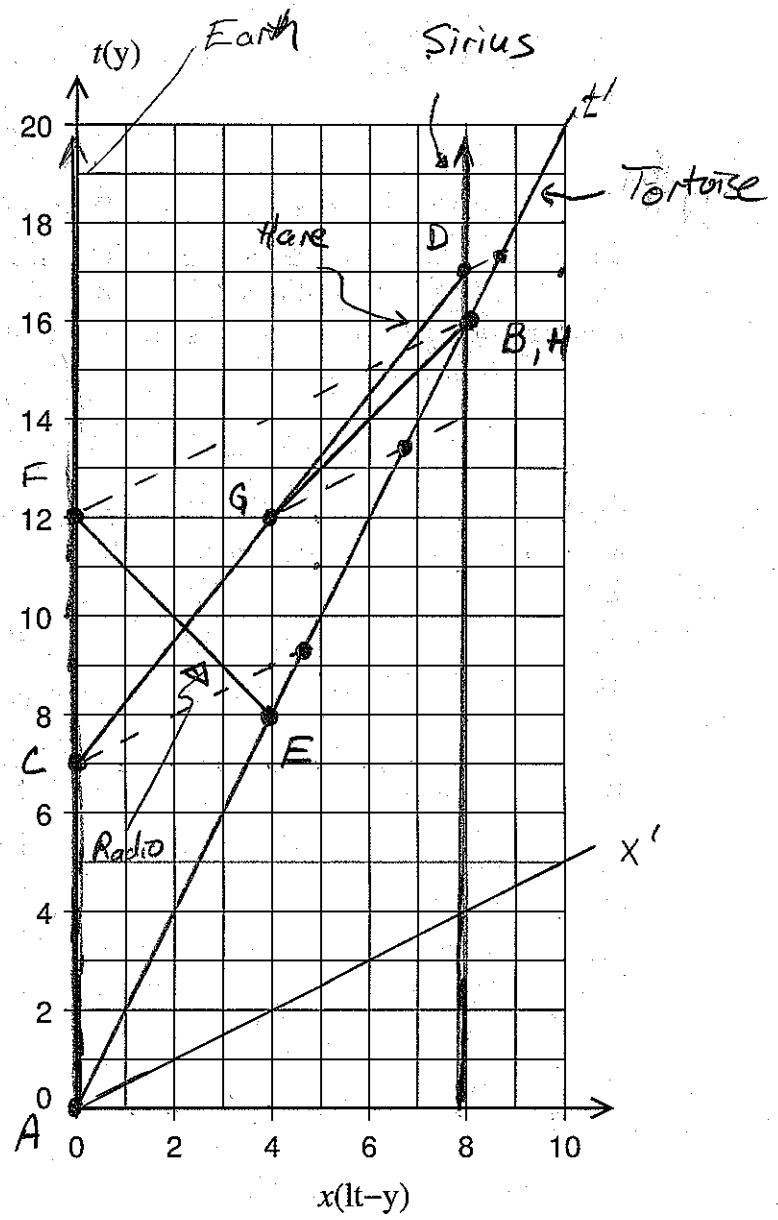
(a) The *Tortoise* blasts off from Earth at time $t = 0$ and plods away at a steady pace of $0.5c$ all the way to Sirius.

(b) When the *Tortoise* reaches the halfway point, it sends a radio signal to the Earth asking the *Millenium Hare* "are you coming?"

(c) The *Millenium Hare* bides its time for 7 years after the *Tortoise* left, and then flies to Sirius at the blazing speed of $0.8c$.

(d) When the *Hare* reaches the halfway point, it sends a radio signal to Sirius saying "I'm gonna catch you!"

(e) Now label all the following events:
**A:** *Tortoise* blasts off, **B:** *Tortoise* arrives, **C:** *Hare* blasts off, **D:** *Hare* arrives, **E:** *Tortoise* sends radio signal, **F:** *Tortoise* radio signal arrives at Earth, **G:** *Hare* sends radio signal, **H:** *Hare* radio signal arrives at Sirius.




(f) Order the events from earliest to latest according to the frame moving at $0.5c$ with the *Tortoise*? Earth Frame: A C E (F,G) (B,H) D

Tortoise Frame: A E C G (B, F, H) D

(g) What is the total travel time according to clocks on board the *Hare*?

$$\Delta t_{Hare} = \Delta t_{proper} = \Delta t_{2\text{-clock}} \sqrt{1 - \frac{v^2}{c^2}}$$

$$= 16\sqrt{1 - 0.5^2}$$

$$= 13.86 \text{ years}$$