Relativistic Energy

Bowling Ball:

- $m = 8\,\text{kg}$
- $v = 10\,\text{m/s}$

Classical Kinetic Energy:

$$K_{\text{cl}} = ?$$

Relativistic Energy:

$$E = ?$$

Relativistic Kinetic Energy:

$$K_{\text{rel}} = ?$$

Relativistic Energy

Bowling Ball ($m = 8\,\text{kg}$ $v = 10\,\text{m/s}$):

Classical Kinetic Energy:

$$K_{\text{cl}} = \frac{1}{2}mv^2 = \frac{1}{2} \times 8 \times 10^2 = 400\ \text{J}$$

Relativistic Energy:

$$E = \frac{mc^2}{\sqrt{1 - v^2/c^2}} = 720000000000000400.00000000000033\ \text{J}$$

Relativistic Rest Energy:

$$E = \frac{mc^2}{\sqrt{1 - {}^2/c^2}} = mc^2 = 8\times(3\times10^8)^2 = 720000000000000000\ \text{J}$$

Relativistic Kinetic Energy:

$$K_{\text{rel}} = E - E_0 = \frac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2 = 400.00000000000033\ \text{J}$$

Binomial expansion

$$(1+\epsilon)^n = 1 + n\epsilon + \frac{n(n-1)}{2!}\epsilon^2 + \frac{n(n-1)(n-2)}{3!}\epsilon^3 + \cdots$$

For $\epsilon \ll 1$:

$$(1+\epsilon)^n \simeq 1 + n\epsilon$$

Examples:

$$(1.02)^3 = (1+0.02)^3 \simeq \text{?}$$

$$\sqrt{1+x} = (1+x)^{1/2} \simeq \text{?}$$

$$\frac{1}{\sqrt{1-x}} \simeq \text{?}$$

Binomial expansion

$$(1+\epsilon)^n = 1 + n\epsilon + \frac{n(n-1)}{2!}\epsilon^2 + \frac{n(n-1)(n-2)}{3!} + \cdots$$

For $\epsilon \ll 1$:

$$(1+\epsilon)^n \simeq 1 + n\epsilon$$

Examples:

$$(1.02)^3 = (1+0.02)^3 \simeq 1 + 3 \times 0.02 = 1.06$$

$$\sqrt{1+x} = (1+x)^{1/2} \simeq 1 + \frac{1}{2}x$$

$$\frac{1}{\sqrt{1-x}} = (1-x)^{-1/2} \simeq \left[1 + \left(-\frac{1}{2}\right)(-x)\right] = 1 + \frac{1}{2}x$$

Expansions in relativity calculations

$$\frac{1}{\sqrt{1 - v^2/c^2}} = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \simeq 1 + \frac{1}{2}\frac{v^2}{c^2}$$

Relativistic Energy:

$$\begin{aligned} E &= \frac{mc^2}{\sqrt{1 - v^2/c^2}} \\ &= mc^2\left(1 + \frac{1}{2}\frac{v^2}{c^2} + \cdots\right) \\ &= mc^2 + \frac{1}{2}mv^2 + \frac{3}{8}\frac{mv^4}{c^2} + \cdots \end{aligned}$$

Relativistic energy/momentum for elementary particles

Proton:

- mass: $m = 938\,\mathsf{MeV}/c^2$
- speed: $u = 0.8c$

Relativistic Rest Energy:

$$E_{\mathrm{rest}} = \frac{mc^2}{\sqrt{1-0^2}} = 938\,\mathsf{MeV}/c^2 \times c^2 = 938\,\mathsf{MeV}$$

Relativistic Energy:

$$E = \frac{mc^2}{\sqrt{1-\left(\frac{0.8c}{c}\right)^2}} = \frac{938\,\mathsf{MeV}}{0.6} = 1563\,\mathsf{MeV}$$

Relativistic Kinetic Energy:

$$K = E - E_{\mathrm{rest}} = 1563\,\mathsf{MeV} - 938\,\mathsf{MeV} = 625\,\mathsf{MeV}$$

Proton Energy as a Function of Speed