Big Topics for Test I translated to Rotation

I. DESCRIBING MOTION (KINEMATICS)

- Position: x
- Angular Position (Angle):

$$\theta = \frac{s}{r} \qquad \text{radians}$$

- Velocity:

$$\overline{v} = \frac{\Delta r}{\Delta t} \quad \text{and} \quad v = \frac{dr}{dt}$$

- Angular Velocity:

$$\overline{\omega} = \frac{\Delta \theta}{\Delta t} \quad \text{and} \quad \omega = \frac{d\theta}{dt} \quad \Rightarrow \quad \omega = \frac{v}{r}$$

- Acceleration:

$$\overline{a} = \frac{\Delta v}{\Delta t} \quad \text{and} \quad a = \frac{dv}{dt}$$

- Angular Acceleration

$$\overline{\alpha} = \frac{\Delta \omega}{\Delta t} \quad \text{and} \quad \alpha = \frac{d\omega}{dt} \quad \Rightarrow \quad \alpha = \frac{a}{r}$$

- Relate graphs of position vs. time, velocity vs. time, and acceleration vs. time for 1-d motion
- Relate graphs of *angle* vs. time, *angular* velocity vs. time, and *angular* acceleration vs. time for motion about a stationary axis.
- Uniform circular motion:

  Magnitude of acceleration: $a = \frac{v^2}{r} = r\omega^2$ Direction: Toward center of circle

- **Non**-uniform circular motion:

  Magnitude of acceleration toward the center: $a_r = \frac{v^2}{r}$

  Magnitude of tangential acceleration: $a_t = \alpha r$

II. DYNAMICS: FORCE AND MOTION, TORQUE AND ROTATION

- Force causes *change* in motion. (Newton's First Law: No force $\Rightarrow$ No change in motion, i.e., constant velocity.)
- Rotational analog of force is torque:

$$\tau = rF\sin\theta$$

- Rotational analog of mass is moment of inertia:

$$I = \sum_i m_i r_i^2 \longrightarrow \int r^2\,dm$$

- Torque causes *change* in rotation. (No torque $\Rightarrow$ No change in angular motion, i.e., constant angular velocity.)
- Newton's Second Law:

$$F_{\text{net}} = ma \quad \text{or, equivalently} \quad F_{\text{net}} = \frac{dp}{dt}$$

- Newton's Second Law for Rotation:

$$\tau = I\alpha$$

NOTE: This is valid for rotation about a fixed axis in an inertial frame, or for rotation about an axis through the center-of-mass in an accelerating frame.

III. WORK AND ENERGY

- The following equations are really statements of the same thing, but with different labels for some of the work done. These are always true.

$$\begin{aligned} \Delta K &= W_{\text{net}} \\ \Delta K &= W_{\text{net,nc}} + W_{\text{net,c}} \\ \Delta K + \Delta U &= W_{\text{net,nc}} \end{aligned}$$

where $\Delta U \equiv -W_{\rm c}$. The last line can also be written as

$$\Delta E = W_{\rm net,nc}.$$

- In the *special case* in which $W_{\rm nc} = 0$, mechanical energy is conserved, or

$$\Delta K + \Delta U = 0.$$

- To use the ideas above you must be able to calculate work.

  $W = \vec{F} \cdot \Delta\vec{r}$ — constant force, linear displacement

  $W = \sum_i \vec{F}_i \cdot d\vec{r}_i$ — path with several segments, each with constant forces, and linear displacements

  $W = \int_{x_1}^{x_2} F(x)\, dx$ — varying force along a one-dimensional path from x_1 to x_2

- For conservative forces the work done around any closed path is zero. ("You get the work back.") For conservative forces the work done between two points is independent of the path between the points.

- Rotational kinetic energy:

$$K_{\rm rot} = I\omega^2$$

- Work done by torques:

$$W_{\rm torques} = \tau\Delta\theta \longrightarrow \int_{\theta_1}^{\theta_2} \tau\, d\theta$$

- Work-Energy Theorem for rotational motion:

$$W_{\rm torques} = \Delta K_{\rm rot}$$