PHYS 211E— Exam #3
Monday, November 24, 2008

Name:________________________________

Show all work for full credit. Answers should have correct units. Your answers should include an explanation of your approach. This explanation can be in the form of a clear mathematical derivation starting from an equation expressing a basic principle of physics, or it can be a brief explanation in words.

Information

$k = 1.38 \times 10^{23}$ J/K	$R = 8.314$ J/K·mol	$(L_f)_{\text{water}} = 334$ kJ/kg
$c_{\text{water}} = 4184$ J/kg·°C	$c_{\text{ice}} = 2050$ J/kg·°C	

1. (a) Consider a sealed container filled with an ideal gas. The container is compressed but the average speed of the gas molecules in the container stays the same. During this process the temperature of the gas (circle one choice)

Decreases **Stays the same** **Increases** **Not enough information**

Justify your answer. Average kinetic energy $\propto T$

Constant average speed
$\Rightarrow$ Constant average kinetic energy
$\Rightarrow$ Constant T

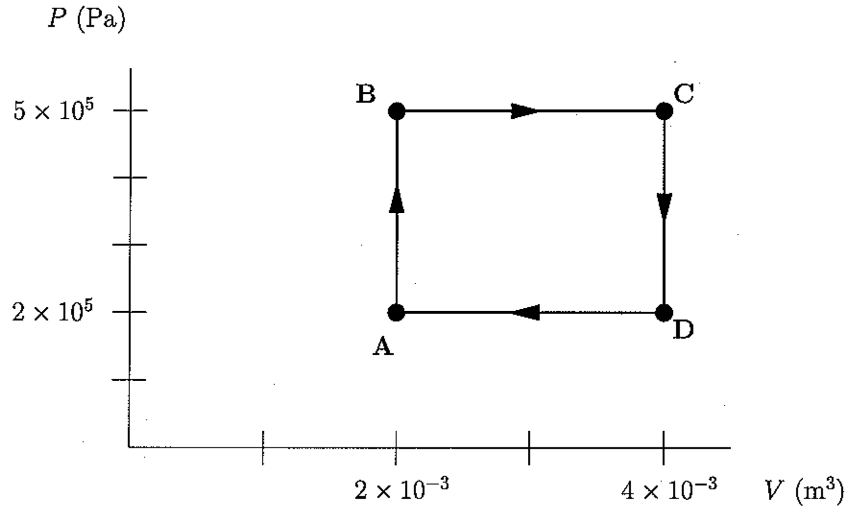
(b) Consider a different sealed container filled with an ideal gas. The container is compressed adiabatically. During this process the temperature of the gas (circle one choice)

Decreases **Stays the same** **Increases** **Not enough information**

Justify your answer. $\Delta U = \cancel{Q_{in}}^{0} - W_{by}$

During compression $W_{by} < 0$
$\Rightarrow \Delta U > 0$
$\Rightarrow \Delta T > 0$

2. The pV diagram shows a fixed amount of an ideal monatomic gas undergoing a cyclic process. The temperature of state **A** is 150 K.



(a) Calculate the temperature of the gas at state **D**.

(b) Calculate the heat added to the gas during the two-step process **A→B→C**.

(c) Calculate the total work done by the gas during one complete cycle **A→B→C→D→A**

a) From ideal gas law you can figure out several things.

$$PV = nRT \Rightarrow \frac{PV}{T} = nR = \text{constant}$$

$$\frac{P_A V_A}{T_A} = \frac{P_D V_D}{T_D} \Rightarrow T_D = \frac{P_D V_D}{P_A V_A} T_A = 2 \times 150$$

$$= 300\,K$$

Can also find $nR = \dfrac{P_A V_A}{T_A} = \dfrac{2\times10^5 \times 2\times10^{-3}}{150} = \dfrac{8}{3}$

$$T_c = \frac{P_c V_c}{P_A V_A} T_A = \frac{5\times4}{2\times2} 150 = 750\,K$$

b) $\Delta U_{A\to C} = Q_{A\to C} - W_{A\to C}$

$$Q_{A\to C} = \Delta U_{A\to C} + W_{A\to C}$$

$$= \frac{3}{2} nR\,(T_C - T_A) + \cancel{W_{A\to B}}^{\,0} + W_{B\to C}$$

$$= \frac{\cancel{3}}{2} \times \frac{8}{\cancel{3}}(750-150) + 5\times10^{5} \times 2\times10^{-3}$$

$$= 2400 + 1000$$

$$= 3400\ \text{J}$$

c) Total work is area enclosed by rectangular cycle.

$$W = 3\times10^{5} \times 2\times10^{-3}$$

$$= 600\ \text{J}$$

3. In an upcoming episode of *CSI: Lewisburg* a villain stabs a man with a 500 g knife made entirely of ice. Fleeing the scene of the crime the villain drops the ice-knife with a temperature of 0° C into an insulated bucket containing 700 g of water with a temperature of 25° C. A crime scene investigator arrives on the scene moments after the bucket and its contents have reached equilibrium. Will the investigator find any evidence of the ice-knife remaining? Support your answer with appropriate calculations.

Assumption A:

Some ice melts, some remains, $T_{final} = 0$.
Solve for m_{melt}

$$m_i c_i (\Delta T)_{ice} + m_{melt} L_f + m_w c_w (\Delta T)_w = 0$$

$$m_i c_i (0+10) + m_{melt} L_f + m_w c_w (0-25) = 0$$

$$m_{melt} = \frac{m_w c_w \times 25 - m_i c_i \times 10}{L_f}$$

$$= \frac{0.7 \times 4184 \times 25 - 0.5 \times 2080 \times 10}{334 \times 10^3}$$

$$= 0.188 \text{ kg}$$

This is a "good" answer; assumption OK.
Some ice-knife remains for CSI team.

Assumption B:

All ice melts; solve for T_f between $0°C$ & $25°C$.

$$m_i c_i \Delta T_i + m_i L_f + m_i c_w (\Delta T')_i + m_w c_w (T_f - 25) = 0$$

Ice increases in temperature from $-10°C \to 0°C$

Ice melts

Water from melted ice increases in temperature from $0°C \to T_f$

Water cools

$$0.5 \times 2050 \times 10 + 0.5 \times 334 \times 10^3 + m_i c_w (T_f - 0) + m_w c_w (T_f - 25) = 0$$

$$10{,}250 + 167{,}000 + T_f (m_i + m_w) c_w - m_w c_w 25 = 0$$

$$T_f = \frac{-177{,}250 + 0.7 \times 4184 \times 25}{(0.5 + 0.7)\, 4184}$$

$$= -20.7\ °C.$$

Not a reasonable answer.

Assumption must be incorrect.

4. The vertical position of a mass hanging on a spring is given by the following function of time:

$$y(t) = 0.2\sin(\overset{\omega}{2.5}t + \pi/4),$$

where position is measured in meters (with positive corresponding to "up") and time is measured in seconds.

(a) Determine the period of oscillation of the mass.

(b) Determine the minimum height of the mass.

(c) Determine the maximum speed of the mass.

a) From expression above $\omega = 2.5$

$$\omega = 2\pi f = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{2.5} = 2.513\ s$$

b) Minimum height when $\sin(2.5t + \pi/4) \rightarrow -1$

$\Rightarrow$ Minimum height $= -0.2\ m$

c) $$\frac{dy}{dt} = 0.2 \times 2.5 \times \cos(2.5t + \frac{\pi}{4})$$

Maximum when $\cos(2.5t + \frac{\pi}{4}) \rightarrow 1$

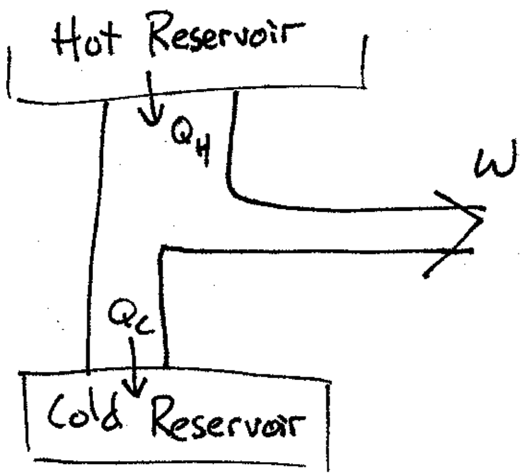
$$\Rightarrow \left.\frac{dy}{dt}\right|_{max} = 0.2 \times 2.5 = 0.5\ m/s$$

5. A steam power plant draws 5000 MW of thermal energy from it's heat source, and it has an efficiency of 30% ($e = 0.3$).

   (a) Calculate the "waste" heat that must dumped into a cold reservoir in one second of operation of the plant.

   (b) Assume that the cold reservoir consists of 8.364 kg of water that increases in temperature from 20° C to 30° C in one second as it cools the steam. Calculate the change in entropy of the water in the cold reservoir in one second.

a)



$$Q_H = 5{,}000 \text{ MW} \times 1 \text{ s} = 5{,}000 \text{ MJ}$$

$$e = \frac{|W|}{|Q_H|} \quad \text{or} \quad |W| = e\,|Q_H| = 0.3 \times 5000 \text{ MJ} = 1500 \text{ MJ}$$

$$|Q_H| = |W| + |Q_c|$$

$$|Q_c| = |Q_H| - |W| = 5{,}000 - 1500 = 3500 \text{ MJ}$$

b)

$$\Delta S = \int_{T_1}^{T_2} \frac{dQ}{T} = \int_{T_1}^{T_2} \frac{mc\,dT}{T} = mc \ln \frac{T_2}{T_1}$$

$$= 8.364 \text{ kg} \times 4184 \text{ J/kg}\cdot{}^\circ\text{C} \ln \frac{303}{293}$$

$$= 1.17 \text{ kJ/K}$$

NOTE: A more realistic statement of this problem would have given 8.364×10^4 kg of water rising 10°C.

The following questions concern the model we have been studying with distinguishable particles in discrete, equally spaced energy levels. The energies of the levels are given by the values 0, ϵ, 2ϵ, 3ϵ, ...

6. System A is in thermal equilibrium; it can be described with the macrostate

$$\{4000, 3800, 3610, 3430, 3258, 3095, \ldots\}$$

System B is also in thermal equilibrium; it can be described with the macrostate

$$\{6000, 5580, 5189, 4826, 4488, 4174, 3882 \ldots\}$$

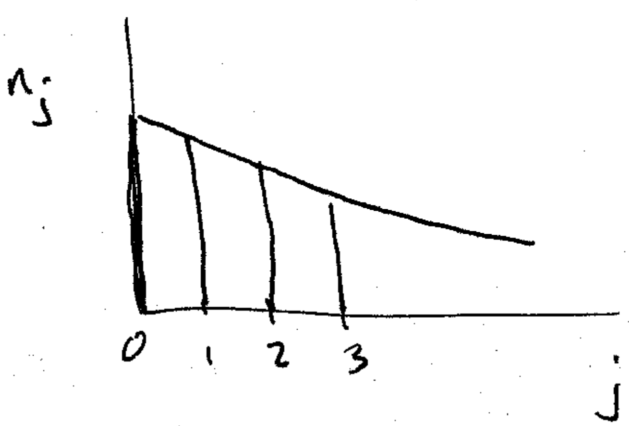
Which system is hotter? Justify your answer.

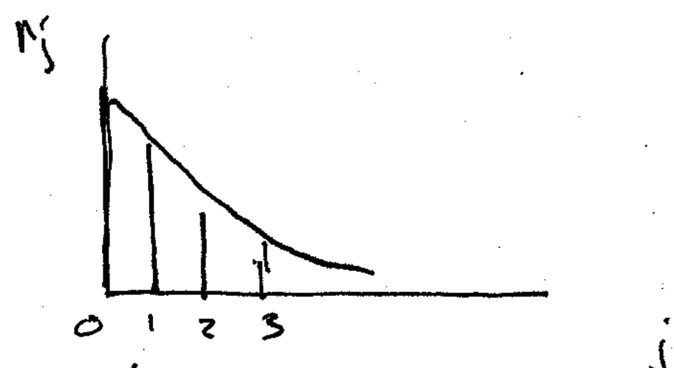
$$\frac{n_1}{n_0} = e^{-\epsilon/kT}$$

System A: $\frac{n_1}{n_0} = \frac{3800}{4000} = 0.95$

System B: $\frac{n_1}{n_0} = \frac{5580}{6000} = 0.93$

$$e^{-\epsilon/kT_A} > e^{-\epsilon/kT_B} \implies T_A > T_B$$





Falls off more rapidly $\Rightarrow$ colder.

7. A gas with 3000 particles is in the macrostate $\{2000, 790, 200, 10, \ldots\}$.

(a) Is this gas in thermal equilibrium? Explain your answer.

$$\frac{n_1}{n_0} = \frac{790}{2000} = 0.395$$

$$\frac{n_2}{n_1} = \frac{200}{790} = 0.253$$

For thermal equilibrium $\frac{n_1}{n_0} = \frac{n_2}{n_1} = \frac{n_3}{n_2} = \cdots$

Not true in this case. Not equilibrium.

(b) A collision between gas particles results in one particle in level 1 (with initial energy ϵ) losing a unit of energy, and a particle in level 2 (with initial energy 2ϵ) gaining a unit of energy. Calculate the **change** in the entropy due to this energy exchange. Does this energy exchange move the system toward equilibrium or away from equilibrium?

Pre-Collision : $\{2000, 790, 200, 10, \ldots\}$

Post-Collision: $\{2001, 789, 199, 11, \ldots\}$

$$\frac{W'}{W} = \frac{\cancel{3000!}}{2001!\, 789!\, 199!\, 11! \ldots} \times \frac{2000!\, 790!\, 200!\, 10! \ldots}{\cancel{3000!}}$$

$$= \frac{790 \cdot 200}{2001 \cdot 11}$$

$$= 7.178$$

$$\Delta S = \Delta(k \ln W) = k \ln \frac{W'}{W}$$

$$= k \ln 7.178$$

$$= k \times 1.971$$

$\Delta S > 0 \Rightarrow$ Moving toward maximum entropy macrostate, or toward equilibrium.