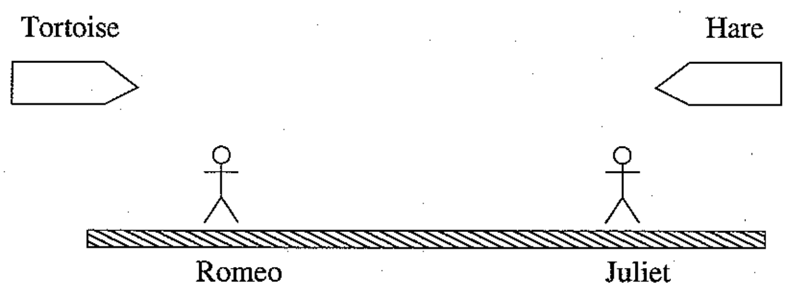
PHYS 211E— Exam #2
Wednesday, October 29, 2008

Name:__________________________________

Show all work for full credit. Answers should have correct units. Your answers should include an explanation of your approach. This explanation can be in the form of a clear mathematical derivation starting from an equation expressing a basic principle of physics, or it can be a brief explanation in words.

1. Romeo and Juliet are standing at rest on the earth, but they are not at the same location. The Millennium Tortoise spacecraft is traveling to the right at a speed of $0.5c$ and the Millennium Hare is traveling to the left at a speed of $0.8c$.



Consider the following four events:

- **Event A:** The Tortoise passes directly overhead Romeo.
- **Event B:** The Hare passes directly overhead Juliet.
- **Event C:** The Tortoise passes directly overhead Juliet.
- **Event D:** The Hare passes directly overhead Romeo.

Circle all correct answers below.

(a) Who measures the proper time between events A and B?

Romeo   Juliet   Hare passenger   Tortoise passenger   None of these observers

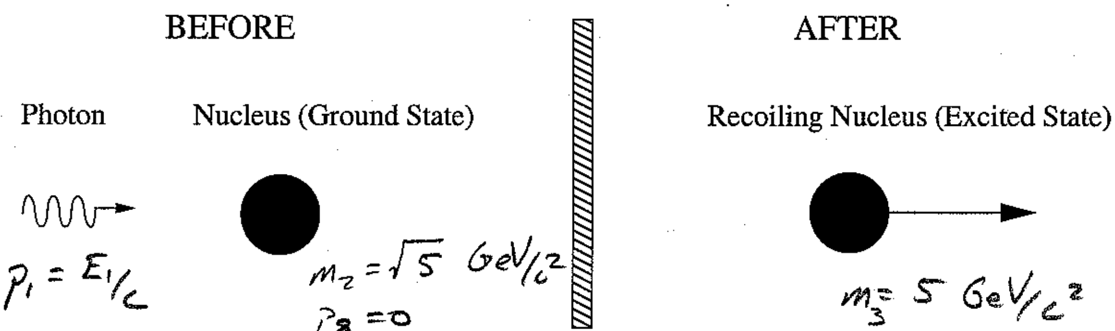
(b) Who measures the proper time between events A and C?

Romeo   Juliet   Hare passenger   Tortoise passenger   None of these observers

(c) Who measures the proper time between events B and C?

Romeo   Juliet   Hare passenger   Tortoise passenger   None of these observers

2. A nucleus with mass $\sqrt{5}\,\mathrm{GeV}/c^2$ in its ground state and is initially at rest. The nucleus absorbs a photon of energy E_1. After absorbing the photon the nucleus is raised to an excited state with mass $5.0\,\mathrm{GeV}/c^2$, and the nucleus recoils with momentum p_3.



$p_1 = E_1/c$

$m_1 = 0$

$m_2 = \sqrt{5}\ \mathrm{GeV}/c^2$

$p_2 = 0$

$E_2 = \sqrt{5}\ \mathrm{GeV}$

$m_3 = 5\ \mathrm{GeV}/c^2$

$E_3^2 = p_3^2 c^2 + (m_3 c^2)^2$

$= p_3^2 c^2 + 25\ \mathrm{GeV}^2$

(a) Calculate the recoil momentum p_3 of the nucleus.

(b) Calculate the energy E_1 of the incident photon.

Conservation of Momentum

$$p_1 + \cancel{p_2}^{\,0} = p_3 \implies p_1 = p_3$$

Conservation of energy

$$E_1 + E_2 = E_3$$

$$p_1 c + \sqrt{5} = \sqrt{p_3^2 c^2 + 25}$$

$$= \sqrt{p_1^2 c^2 + 25}$$

$$(p_1 c + \sqrt{5})^2 = p_1^2 c^2 + 25$$

or $$\cancel{p_1^2 c^2} + 5 + 2\sqrt{5}\, p_1 c = \cancel{p_1^2 c^2} + 25$$

$$p_1 c = \frac{20}{2\sqrt{5}} \implies p_1 = 2\sqrt{5}\ \mathrm{GeV}/c = p_3$$

$$E_1 = p_1 c = 2\sqrt{5}\ \mathrm{GeV}$$

3. Juliet is traveling in her relativistic gondola at $0.8c$ as she passes by Romeo who is standing on the bank of a canal. Romeo and Juliet both watch the physics experiment described in problem 2 on the previous page. Romeo and Juliet agree on some aspects of the experiment, but not on others. Circle the appropriate responses below.

   (a) Speed of the nucleus before the absorption

   R&J **Agree**   R&J **Disagree**   Not enough information to tell

   (b) Speed of the photon before the absorption

   R&J **Agree**   R&J **Disagree**   Not enough information to tell

   (c) Mass of the recoiling nucleus

   R&J **Agree**   R&J **Disagree**   Not enough information to tell

   (d) Difference in the masses of the nucleus before and after the absorption.

   R&J **Agree**   R&J **Disagree**   Not enough information to tell

   (e) Momentum of the nucleus before the absorption

   R&J **Agree**   R&J **Disagree**   Not enough information to tell

   (f) Total momentum before the absorption

   R&J **Agree**   R&J **Disagree**   Not enough information to tell

   (g) Difference in the total momentum before and after the absorption

   R&J **Agree**   R&J **Disagree**   Not enough information to tell

4. The Apollo 137 mission sends a Rocket at speed $0.75c$ from Earth to Alpha-Centauri which is 4.5 lt-y away. An astronomer on Alpha Centauri looks through a telescope and observes the light flash due to the Rocket launching from Earth. After traveling a distance of 1.5 lt-y in the Earth frame the astronauts in the Rockey send a radio signal back to Earth announcing that they need food. (Radio waves travel at the speed of light.) In the spacetime diagram below:

   (a) Sketch the world line for Earth.

   (b) Sketch the world line for Alpha-Centauri.

   (c) Sketch the world line for the light from the rocket launch.

   (d) Sketch the world line for the radio signal back to Earth.

   (e) Label the events

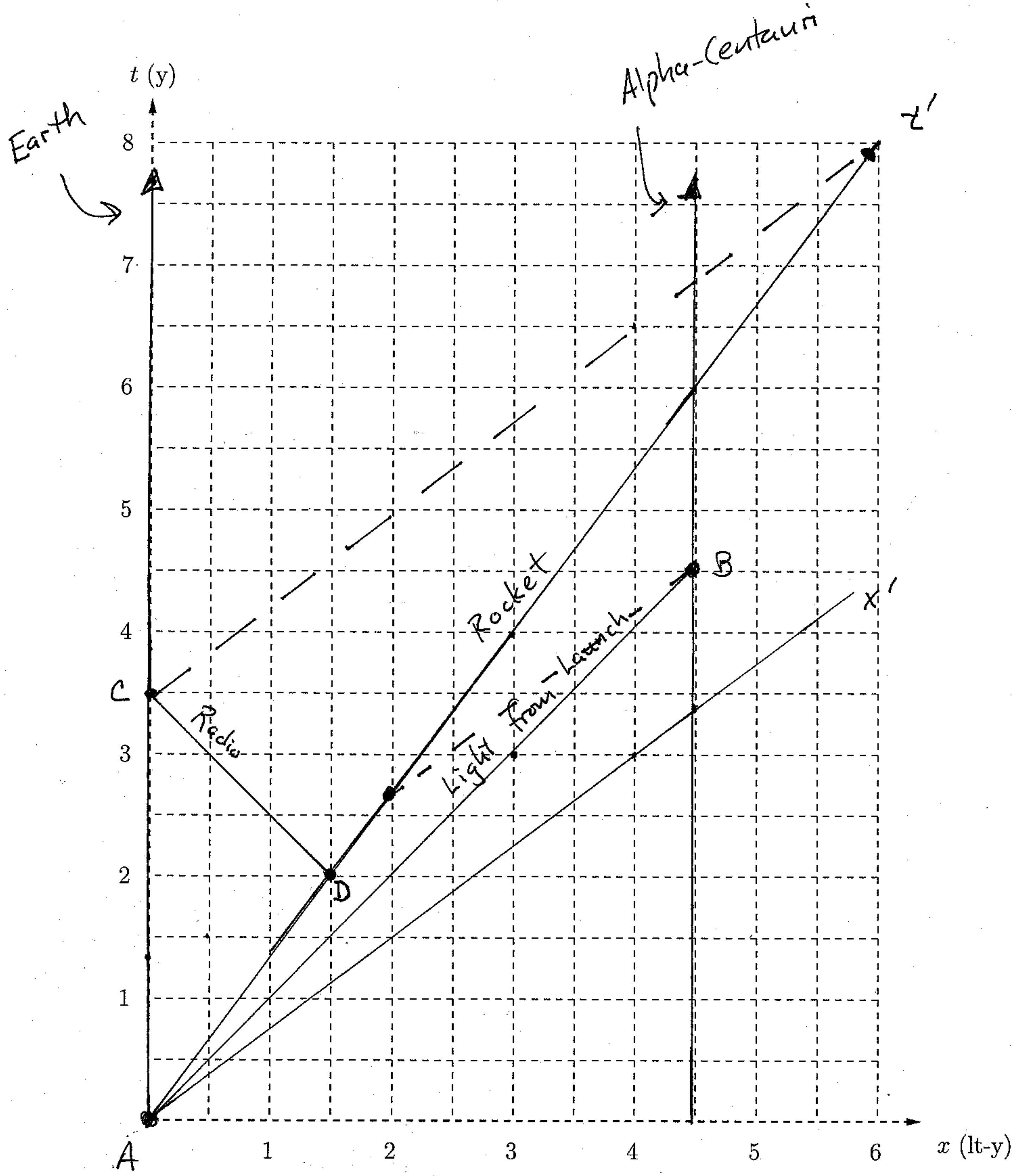
      i. A: Launch

      ii. B: Astronomer sees launch

      iii. C: Earth receives signal for food

      iv. D: Rocket sends signal for food

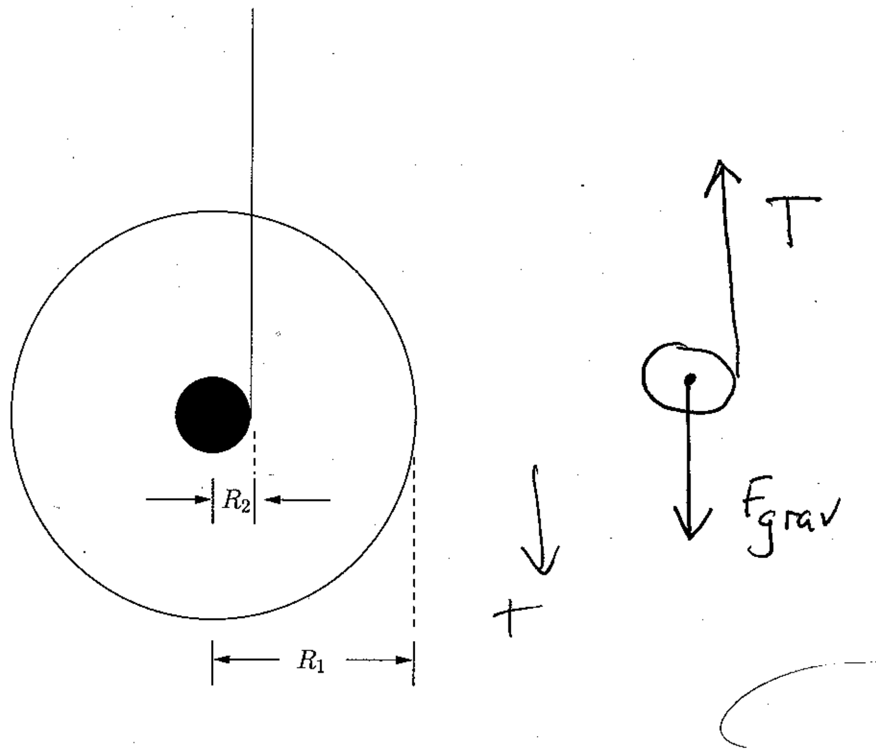
   (f) Order these events as observed in the astronaut's frame.



t (y)
Earth
Alpha-Centauri
t'
x'
Rocket
Light from Launch
Radio
A
B
C
D
x (lt-y)
8
7
6
5
4
3
2
1
1
2
3
4
5
6

Order A D B C

5. Consider a yo-yo with a mass $m = 0.05\,\text{kg}$, outer radius $R_1 = 3\,\text{cm}$, rotational inertia $I = 1.6 \times 10^{-5}\,\text{kg}\cdot\text{m}^2$ and the radius of the axle on which the string is wound is $R_2 = 0.6\,\text{cm}$. The string is very thin thread that does not affect the radius of the axle. After winding up the string you release the yo-yo from rest while holding the top of the string stationary.



(a) Determine the magnitude of the angular acceleration of the yo-yo as it falls.

(b) Determine the angular velocity of the yo-yo 1 second after it is released.

$$\tau_{net} = I\alpha \qquad F_{net} = ma$$

$$\tau_{grav} + \tau_T = I\alpha \qquad mg - T = ma = mR_2\alpha$$

← Torques calculated about center-of-mass

$$0 + TR_2 \sin 90^\circ = I\alpha \qquad mg - \frac{I\alpha}{R_2} = mR_2\alpha$$

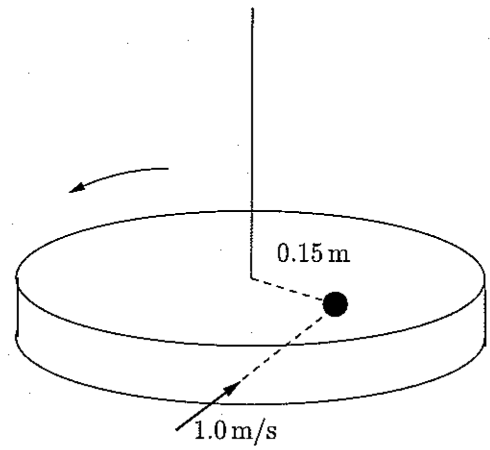
$$\Rightarrow T = \frac{I\alpha}{R_2}$$

$$\alpha\left(mR_2 + \frac{I}{R_2}\right) = mg$$

$$\alpha = \frac{mg}{mR_2 + I/R_2}$$

$$= \frac{0.05 \times 9.8}{0.05 \times 0.006 + 1.6\times10^{-5}/0.006}$$

6. A turntable with a radius $R = 0.25\,\text{m}$ and rotational inertia $I = 0.5\,\text{kg}\cdot\text{m}^2$ is rotating about a frictionless vertical axis with an angular velocity of $1.5\,\text{rad/s}$. A wad of clay with mass $30\,\text{g}$ is tossed onto the turntable and sticks to the surface $0.15\,\text{m}$ from the center of the table. The clay hits with a horizontal component of velocity of $1.0\,\text{m/s}$, and the velocity is directed at right angles to the turntable's radius so as to increase the rotation rate. Find the final angular speed of the turntable after the clay sticks to the table.



Conservation of angular momentum:

$$L_{before} = L_{after}$$

$$L_{clay} + L_{table\ before} = L_{clay+table\ after}$$

$$+ mvr \sin 90^\circ + I_{table}\,\omega_{before} = I_{table+clay}\,\omega_{after}$$

$$\Rightarrow \quad \omega_{after} = \frac{m v r + I_{table}\,\omega_{before}}{I_{table+clay}}$$

$$= \frac{0.03 \times 1 \times 0.15 + 0.5 \times 1.5}{(0.5 + 0.03 \times 0.15^2)}$$

(0.5 ↑ I_{table}; 0.03×0.15^2 ↑ $I_{clay} = mr^2$)

$$= 1.507\ \text{rad/s}$$