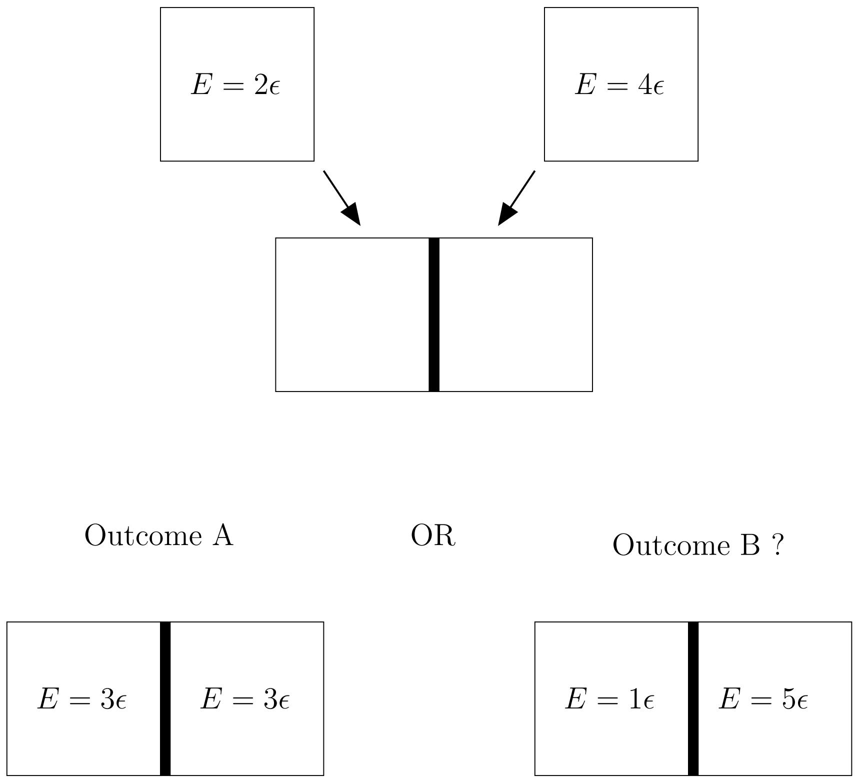
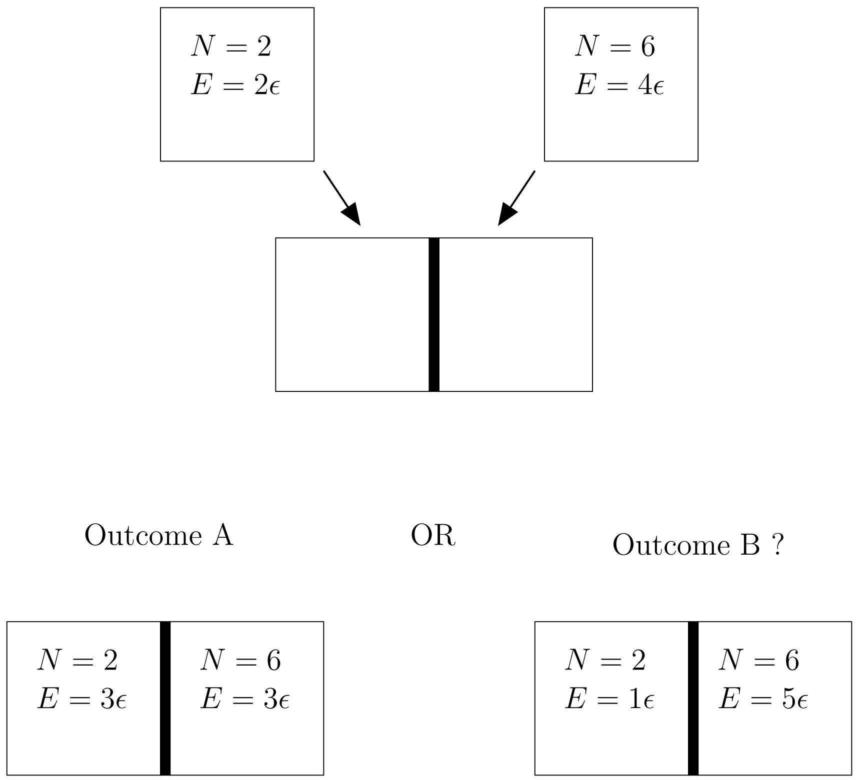
Energy/Heat Flow and Entropy (Corrected)

Consider the following scenario: A box whose contents have an energy 2ϵ is brought into thermal contact with a box whose contents have an energy 4ϵ. After an exchange of 1 unit of energy, which illustrated outcome is more likely? (This example uses the model of equally spaced energy levels discussed in Chapter 7 of the Supplementary Reading.)



Both outcomes conserve energy, and the natural response to this question is that Outcome A is more likely. But this is not really a fair question — there is not enough information given to answer this question. To see why this might be the case, consider a small volume of milk, say $1\,\mathrm{cm}^3$, being added to a larger volume of cocoa, say $100\,\mathrm{cm}^3$. If the large volume of cocoa has a temperature of $90^\circ\,\mathrm{C}$, and the small volume of milk has temperature that is close to that of the cocoa it should be obvious that the larger mass of cocoa has more internal energy than small mass of milk. But does this mean that heat will flow from the cocoa to the milk? Not necessarily. You need to know the *temperature* of the milk before you know which way the thermal energy (i.e., heat) will flow, and temperature is not the same thing as internal energy. In the example above you have no way of estimating the relative temperature of the two systems from the given information.

Let's add some information to the scenario given above: let the box on the left contain 2 particles, and the box on the right contain 6 particles.



Now let's analyze the two outcomes by calculating the number of microstates associated with each of them.

Outcome A			
$N = 2$, 3ϵ system		$N = 6$, 3ϵ system	
Macrostates	W	Macrostates	W
$\{1,0,0,1,0,\ldots\}$	2	$\{5,0,0,1,0,\ldots\}$	6
$\{0,1,1,0,\ldots\}$	2	$\{4,1,1,0,0,\ldots\}$	30
		$\{3,3,0,\ldots\}$	20
	4		56
$(W_{\text{total}})_{\text{A}} = 4 \times 56 = 224$			

Outcome B			
$N = 2$, 1ϵ system		$N = 6$, 5ϵ system	
Macrostates	W	Macrostates	W
{1,1,0,...}	2	{5,0,0,0,0,1,0,...}	6
		{4,1,0,0,1,0,...}	30
		{4,0,1,1,0,...}	30
		{3,2,0,1,0,...}	60
		{3,1,2,0,...}	60
		{2,3,1,...}	60
		{1,5,0,...}	6
	2		252
$(W_{\text{total}})_{\text{B}} = 2 \times 252 = 504$			

These calculations show that Outcome B is more likely — the energy is more likely to flow from the high energy system to the low energy system. This is makes a little more intuitive sense when you consider that in the original state the system on the left had an average energy per particle of ϵ, while the system on the right had an average energy per particle of $\frac{2}{3}\epsilon$. Energy flowed from a system with a higher energy per particle to a system with a lower energy per particle. In other words, the energy flowed from the hotter system to the colder system.