ENGR 695 Advanced Topics in Engineering Mathematics Fall 2024

Lab #7: Animations of Vibrating Strings

Introduction

In this lab session, you will examine in more detail the solution of the wave equation using the separation of variables method as it is applied to analyzing the vibration of a string. You will have the opportunity to code an initial condition that illustrates the superposition of two counterpropagating traveling waves, and you will code a second one that you select yourself. You will also examine the speed of propagation of the waves.

Theoretical Background

The one-dimensional wave equation and the boundary conditions and initial conditions that describe the ideal vibration of a string stretched between two fixed points are given by

$$v^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \qquad 0 \le x \le L \quad \text{and} \quad t \ge 0$$

$$u(0,t) = 0, \quad u(L,t) = 0, \quad u(x,0) = f(x), \quad \text{and} \quad \left.\frac{\partial u(x,t)}{\partial t}\right|_{t=0} = g(x),$$

where the dependent variable u represents the displacement of the string at any location and instant in time. The function $f(x)$ specifies the displacement of the string at time $t = 0$. This initial condition would be used if the string were not moving at $t = 0$ but instead were held with some starting displacement and then suddenly released. The function $g(x)$ specifies the displacement velocity of the string at time $t = 0$. It describes the initial movement of the string, if any, caused by an action such as striking it with a mallet.

The vibrational waves on the string are transverse waves, which means that the string is displaced from its resting condition (a straight line between the fixed attachment points at the boundaries) in a direction transverse (perpendicular) to the x-axis. That is, the u-axis is perpendicular to the x-axis. This means that u should be expressed in a length unit.

If the separation of variables method is applied to the problem, the resulting solution is

$$u(x,t) = \sum_{n=1}^{\infty} \left[A_n \sin\left(\frac{n\pi x}{L}\right) \cos\left(\frac{n\pi v t}{L}\right) + B_n \sin\left(\frac{n\pi x}{L}\right) \sin\left(\frac{n\pi v t}{L}\right) \right],$$

$$\text{where} \quad A_n = \frac{2}{L}\int_0^L f(x) \sin\left(\frac{n\pi}{L} x\right) dx \quad \text{and} \quad B_n = \frac{2}{n\pi v}\int_0^L g(x) \sin\left(\frac{n\pi x}{L}\right) dx .$$

This solution applies only to the specific problem outlined above. If the boundary conditions or initial conditions were to change, so would the solution.

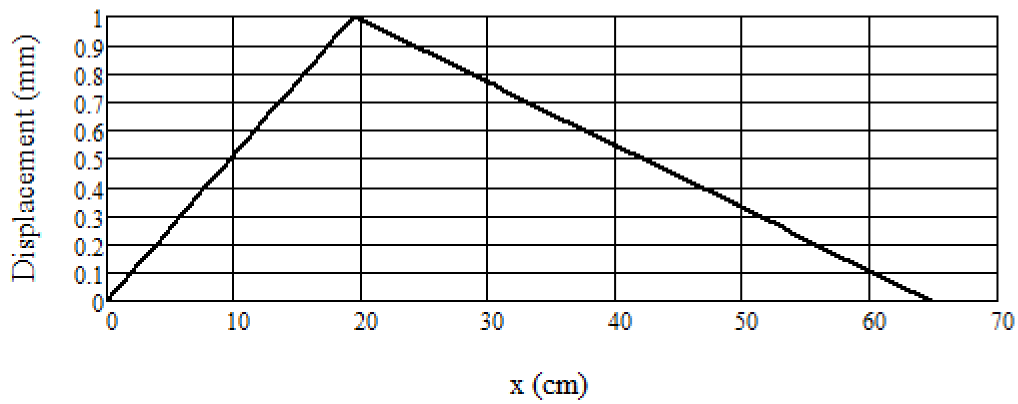
Procedure

- Download the following *Matlab* m-files, which are available at the course Moodle site. You should set up a separate folder to contain them.

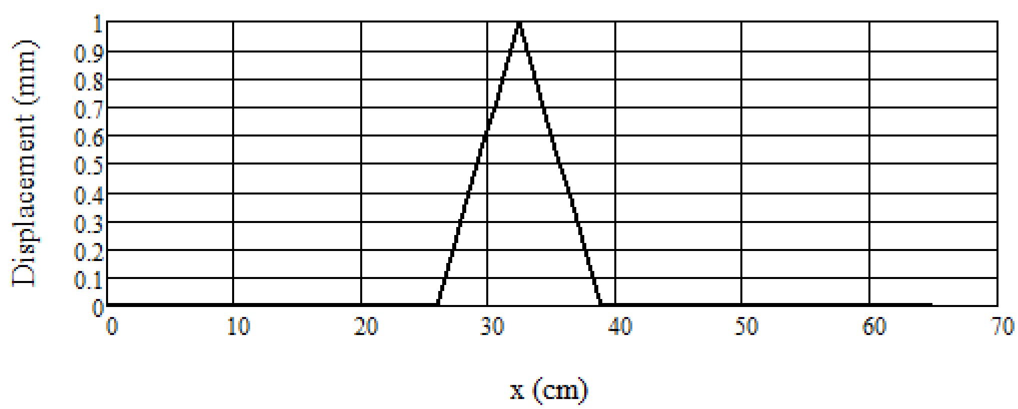
  `VibratingStringExample.m` – primary script containing most of the required code
  `fsin.m` – function that defines the integrand used to find the coefficients $\{A_n\}$
  `gsin.m` – function that defines the integrand used to find the coefficients $\{B_n\}$
  `f_lab.m` – function that defines the initial displacement of the string $f(x)$
  `g_lab.m` – function that defines the initial transverse velocity of the string $g(x)$

  Change the names of `f_lab.m` and `g_lab.m` to `f.m` and `g.m`, respectively, to allow the code to execute properly.

  Try running the scripts without modification to make sure that you have downloaded everything and that *Matlab* is pointing to the correct folder. Look over the source code and try to understand in general how it works. The default initial displacement of the string should have the shape shown below.



- After you are confident that the script is working, define a new displacement function $f(x)$ that has the shape shown below. The locations that correspond to the start, peak, and stop points of the triangular section are $x = 0.4L$, $x = 0.5L$, and $x = 0.6L$, respectively. Tips for writing the necessary code will be provided during the lab session. The other initial condition should remain $g(x) = 0$.



- Run the script with the new *f*(*x*) function and note carefully what you observe. Remember that the solution is a superposition of a large number of sinusoidal eigenfunctions in space multiplied by sinusoidal functions in time. That is, the solution is technically a standing wave. However, this example clearly illustrates that a standing wave can be thought of as two traveling waves propagating in opposite directions.

- Estimate the wave propagation speed based on what you observe in the *Matlab*-generated Figure 2 (the main animation window) and then verify that it is close to the numerical value given in the window's heading. Briefly explain in comments added to your `f.m` file how you determined the speed. Some of the important parameters that you might need are given below:

  F_T = 58.3 N: string tension (represented by T in the textbook)
  ρ = 0.3203 g/m: mass per unit length of the string
  L = 64.77 cm: length of the string

- Devise another new displacement function *f*(*x*) that you select yourself and write the code to implement it. If you can, try to highlight an interesting behavior or a particular mathematical characteristic of the solution. Alternatively, you may define a non-zero *g*(*x*) function instead of *f*(*x*). If you do, set *f*(*x*) = 0. Briefly explain using comments added to your `f.m` file why you chose the function and what the solution associated with it illustrates.

<u>Lab Work Submission and Scoring</u>

Save a copy of your edited script `f.m` (and `g.m` if you edited it as well). Change the name(s) to `LName_f_Lab7_fa24.m` (and `LName_g_Lab7_fa24.m`, if necessary), where `LName` is your last name. Add comments to the script(s) that 1) explain how you determined the wave propagation speed based on your observations of the animation and that 2) explain your reason for selecting your chosen *f*(*x*) or *g*(*x*) function, and then e-mail the file(s) to me.

Your score will be based primarily on your submitted *Matlab* script and will be determined according to the rubric posted on the Laboratory page at the course web site.

You may submit your m-file at any time before 11:59 pm on Friday, Nov. 8. If the file is submitted after the deadline, a 10% score deduction will be applied for every 24 hours or portion thereof that it is late (not including weekend days) unless extenuating circumstances apply. No credit will be given five or more days after the deadline.