ENGR 695 Advanced Topics in Engineering Mathematics Fall 2024

Selected Answers to HW #5

Include explanatory text and intermediate calculations in your solutions. You will not receive credit for merely repeating an answer given here without supporting work.

If an answer is not provided below, it is either because the solution is trivial or because disclosure of the answer would reveal too much of the solution.

It is possible that one or more of the answers given below are incorrect. There is a trade-off between speed and accuracy. If you suspect that an answer below is incorrect, please let me know as soon as possible.

1. $$u(x,t) = A_0 e^{-ht} + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi x}{L}\right) e^{-\left(\frac{kn^2\pi^2}{L^2}+h\right)t}$$

with $A_0 = \frac{1}{L}\int_0^L f(x)\,dx$ and $A_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$

2. $$u(x,t) = \sum_{n=1}^{\infty} A_n \left[e^{-\beta t}\cos(q_n t) + \frac{\beta}{q_n} e^{-\beta t}\sin(q_n t)\right]\sin(nx), \quad \text{where} \quad q_n = \sqrt{n^2-\beta^2},$$

with $A_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin(nx)\,dx$

3. $$u(r,t) = \sum_{n=1}^{\infty} A_n J_0(\alpha_n r) e^{-k\alpha_n^2 t} \quad \text{with} \quad A_n = \frac{\int_0^c f(r) J_0(\alpha_n r)\, r\, dr}{\left\| J_0(\alpha_n r)\right\|^2}$$

4. $$u_{i,j,n+1} = C_x\left(u_{i+1,j,n} + u_{i-1,j,n}\right) + C_y\left(u_{i,j+1,n} + u_{i,j-1,n}\right) + \left(1 - 2C_x - 2C_y\right)u_{i,j,n},$$

where coefficients C_x and C_y are defined in the problem statement