# ELEC 470 Communication and Information Systems Fall 2014

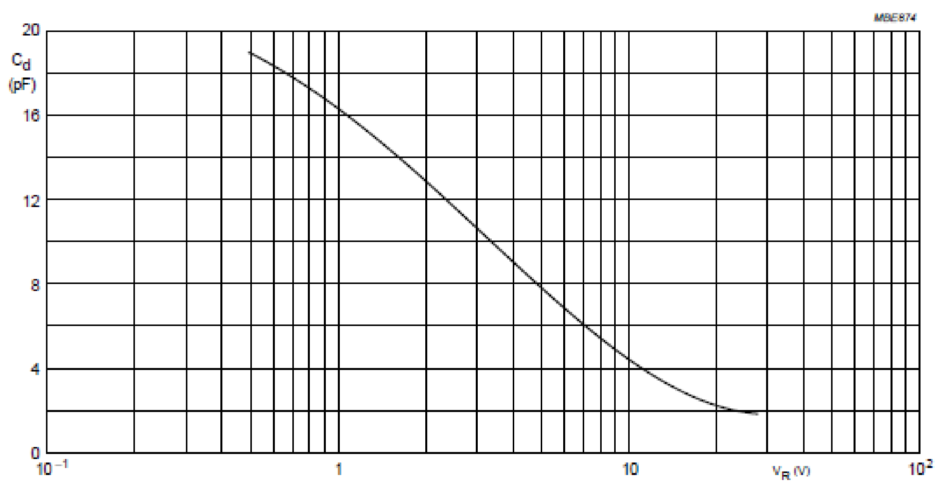
**Homework Assignment #4 – due in class on Wednesday, Nov. 19, 2014**

***Instructions, notes, and hints*:**

You may make reasonable assumptions and approximations in order to compensate for missing information, if any. Provide the details of all solutions, including important intermediate steps. You will not receive credit if you do not show your work.

***Assignment*:**

1. An FM modulator for WVBU is to be designed using the direct method with a varactor diode (voltage-controlled capacitance) in parallel with a 100-nH inductor and a 20-pF capacitor. The desired carrier frequency is 90.5 MHz, and the desired peak frequency deviation is 75 kHz. An NXP Semiconductors type BB135 varactor diode will be used, which has the capacitance vs. reverse bias voltage relationship shown in the figure below. What average reverse bias voltage $V_R$ and what peak-to-peak modulating voltage $v_{r,pk}$ must be applied to the varactor to achieve the specifications?




2. A colleague has come up with an idea for generating an FM signal using a variation of the direct method. She wonders if the variable pressure of sound waves could be used to modulate the relative permittivity of a dielectric foam inserted between the plates of a capacitor. The capacitance of a parallel-plate capacitor is given by

$$C = \frac{\varepsilon_r \varepsilon_o A}{d},$$

where $\varepsilon_r$ is the relative permittivity of the dielectric, $\varepsilon_o$ is the permittivity of free space ($8.854 \times 10^{-12}$ F/m), $A$ is the area of one plate, and $d$ is the separation distance between the plates.

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To test the idea, you build a capacitor using a 0.1-mm thick foam dielectric that has a nominal (unmodulated) relative permittivity of 2.0. Each plate is 1 cm square (i.e., 1 cm × 1 cm). To generate an FM signal with a carrier frequency of 90.5 MHz and a peak deviation of 75 kHz, what is the required value of the inductor that would be paired with the capacitor, and what maximum variation in the relative permittivity $\Delta\varepsilon_r$ about its nominal value would be required?

3. An interesting approach for demodulating FM signals is to approximate a time derivative using a delay line as shown in the figure below. (An FM signal can be demodulated by passing it through differentiator and then an envelope detector.) Recall that a first-order derivative in the time domain corresponds to multiplication by $j\omega$ in the frequency domain. The idea is that a derivative can be approximated by a finite difference, defined as

$$y(t) = \frac{dx(t)}{dt} \approx \frac{x(t) - x(t-\tau)}{\tau},$$

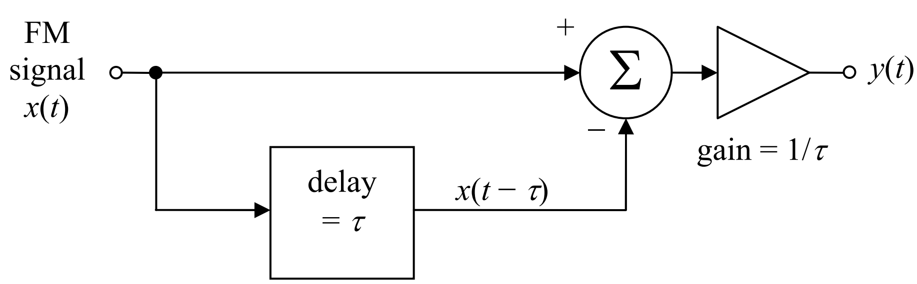
where $\tau$ is a short time delay. Taking the Fourier transform of both sides of the expression above yields

$$Y(\omega) = \frac{1}{\tau}\left[X(\omega) - e^{-j\omega\tau}X(\omega)\right] = \frac{X(\omega)}{\tau}\left(1 - e^{-j\omega\tau}\right).$$

If $\omega t << 1$, then the relationship can be approximated as

$$Y(\omega) \approx \frac{X(\omega)}{\tau}\left[1 - (1 + j\omega\tau)\right] = j\omega X(\omega).$$

If this approach were used to demodulate the WVBU signal at 90.5 MHz using $\omega t = 0.1$, how long would the delay line have to be, and what would be the required gain in dB of the amplifier that follows the summing junction? Assume $x(t)$ is a voltage signal, and assume that waves propagate at the speed of light in free space ($3 \times 10^8$ m/s) along the delay line.

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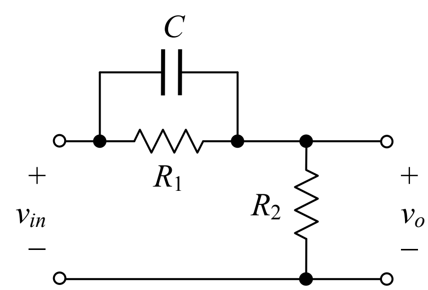
4. A frequency division multiplexing (FDM) scheme uses 100 adjacent audio channels, each of which is 3 kHz wide (2.4 kHz plus a 600 Hz "guard band"), for a total bandwidth of 300 kHz. Each channel contains a SSB-SC signal (USB), and the modulating audio signals have equal amplitude. The multiplexed signal feeds an FM modulator with a peak frequency deviation of 1 MHz.
    a. Determine the bandwidth of the composite FM signal.
    b. If no pre-emphasis/de-emphasis is used in the system, what is the degradation in signal-to-noise ratio (in dB) from the first channel centered at 1.5 kHz to the 100$^{th}$ channel centered at 299.5 kHz? Assume white noise at the receiver's input, and assume that the noise is roughly uniform over a given 3-kHz channel.
    c. Repeat part *b* if PM (phase modulation) were used.

5. To flatten the noise performance of the FDM system described in the previous problem, the pre-emphasis filter shown below will be added to the transmitter just before the FM modulator stage. The filter's lower and upper cut-off frequencies are to be $f_1$ = 2.1 kHz and $f_2$ = 400 kHz, respectively. Assume that the circuit stage following the filter has a high enough input impedance not to load down the filter. The designer would like to use a precision 1000-pF capacitor in the circuit. Find the values of $R_1$ and $R_2$ to meet the specification.




6. A direct digital synthesizer (DDS) has been designed with a clock frequency of 10 MHz. The sine wave look-up table (ROM) uses 24-bit addressing, but the sine wave values stored in the ROM are truncated to 16 bits. The ROM stores only the first quarter cycle of sine wave values (corresponding to phase values from 0 to $0.5\pi$ radians), so two of the 24 addressing bits are used to determine the phase quadrant. [The phase accumulator section of the DDS system is not the register that collects the sine wave values as erroneously mentioned in the lecture on Friday, Nov.14. It is the group of circuit stages that determine the address applied to the ROM.]
    a. To what value should the phase increment *M* (i.e., the ROM address step size) be set by the user to produce an output signal with a frequency as close as possible to 1.5 MHz?
    b. What is the tuning frequency increment of the DDS system? That is, how much does the output frequency change for a change of ±1 in the value of *M*?

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7. The diagram below shows a type of *image rejection mixer*. It is actually a collection of mixers, phase shifters, and filters, and its purpose is to eliminate the presence of the image signal in a receiver. For the architecture shown, which frequency ($\omega_1$ or $\omega_2$) would be downconverted to the IF, and which would be rejected (i.e., which is the desired frequency, and which is the image)? The local oscillator frequencies are related by $\omega_x > \omega_y$. Furthermore, $\omega_1 > \omega_y$ and $\omega_2 > \omega_y$. All of the low-pass filters are designed to eliminate the sum product outputs of the mixers they follow. Provide the details of your solution. *Note*: If the diagram below does not make sense to you, see the supplemental notes on mixers for an example of its interpretation. One or more of the following trigonometric identities might be helpful:

$$\cos(-x) = \cos x \qquad \sin(-x) = -\sin x \qquad \cos(x + 90°) = -\sin x \qquad \cos(x - 90°) = \sin x$$

An advantage of this type of mixer is that a highly selective band-pass filter is not required at the input of the receiver. However, in practice it is difficult to ensure that the amplitudes of the signals at points $v_e$ and $v'_e$ (i.e., at the inputs of the summing junction) are equal. If the amplitudes do not match, then only partial cancellation of the image signal is achieved. Thus, at least a moderate amount of input filtering is still required. Another obvious disadvantage is that four mixers, four low-pass filters, and two LOs are required. Nevertheless, the cost and complexity might be tolerable if otherwise a complicated tunable input filter would have been needed.