Homework Assignment #4 – due via Moodle at 11:59 pm on Wednesday, Mar. 5, 2025

Instructions, notes, and hints:

You may make reasonable assumptions and approximations to compensate for missing information, if any. Provide the details of all solutions, including important intermediate steps. You will not receive credit if you do not show your work.

It might be necessary to use reasonable approximations or assumptions to solve some of these problems, especially if critical information is missing. In those cases, your answer might differ from the posted answer by a significant margin. That's okay. If you justify any approximations that you make, you will be given full credit for such answers.

The first set of problems will be graded and the rest will not be graded. Only the graded problems must be submitted by the deadline above. Do not submit the ungraded problems.

Graded Problems:

1. In one of the ungraded problems from the previous homework set, the formula shown below left was used to convert a Z-parameter matrix to its equivalent S-parameter matrix, where Z_0 is the reference impedance. Starting with that conversion formula, derive the formula below right that can be used to convert S-parameters to the equivalent Z-parameters. Square brackets indicate matrix quantities; $[Z_0]$ is a diagonal matrix in which all of the entries on the main diagonal are Z_0 (50 Ω in this case), and $[I]$ is the identity matrix. The quantity Z_0 without brackets is a scalar. Thus, $[Z_0] = Z_0 [I]$.

$$[S] = \left([Z]+[Z_0]\right)^{-1}\left([Z]-[Z_0]\right) \quad \rightarrow \quad [Z] = Z_0\left([I]+[S]\right)\left([I]-[S]\right)^{-1}$$

2. The pi network shown below looks like an impedance matching network, but it is actually a bandpass filter. The passband is centered on the design frequency of 4.0 MHz. Take advantage of symmetry to find all four S-parameters at the design frequency for a reference impedance of 50 Ω. Then express the magnitudes of the S-parameters in decibels. Remember that the S-parameters as defined in class are voltage ratios. You may use any analysis method that you wish to find the port voltages, but nodal analysis will calculate both port voltages simultaneously for a given source and load configuration.
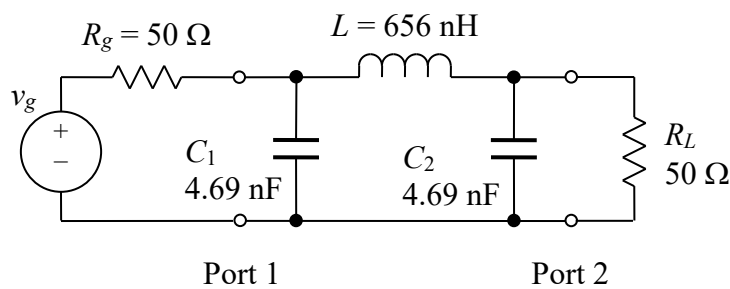
3. Repeat the previous problem for a reference impedance of 75 Ω.

Ungraded Problems:

The following problems will not be graded. However, you should attempt to solve them on your own and then check the solutions. Try not to give up too quickly if you struggle to solve any of them. Move on to a different problem and then come back to the difficult one after a few hours.

1. Use the results from Graded Problem 2 above to find the equivalent Z matrix for the network using the conversion formula below. The $[S50]$ matrix contains the S-parameters associated with the reference impedance of 50 Ω. The scalar value Z_{01} that pre-multiplies the matrix part of the expression is the reference impedance; that is, $Z_{01} = 50$ Ω. $[I]$ is the identity matrix.

$$[Z] = Z_{01}([I]+[S50])([I]-[S50])^{-1}$$

2. Verify the results in the previous problem by applying circuit analysis to find the Z parameters. Remember that Z parameters can be determined by applying a test current source to one port and an open circuit to the other port and finding the resulting port voltages. Each Z parameter is the ratio of the appropriate port voltage to the test current.

3. Use the results from the previous two problems to calculate the equivalent S matrix for a 75 Ω reference impedance from the Z matrix, and compare the results to those of Graded Problem 3. The $[S75]$ matrix contains the new S-parameters. Matrix $[Z_{02}]$ is a diagonal matrix in which all of the entries on the main diagonal are equal to the new reference impedance of $Z_{02} = 75$ Ω.

$$[S75] = ([Z]+[Z_{02}])^{-1}([Z]-[Z_{02}])$$