

**Homework Assignment #3 – due via Moodle at 11:59 pm on Saturday, Mar. 1, 2025**

**Instructions, notes, and hints:**

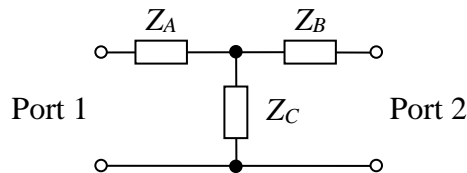
You may make reasonable assumptions and approximations to compensate for missing information, if any. Provide the details of all solutions, including important intermediate steps. You will not receive credit if you do not show your work.

It might be necessary to use reasonable approximations or assumptions to solve some of these problems, especially if critical information is missing. In those cases, your answer might differ from the posted answer by a significant margin. That's okay. If you justify any approximations that you make, you will be given full credit for such answers.

The first set of problems will be graded and the rest will not be graded. Only the graded problems must be submitted by the deadline above. Do not submit the ungraded problems.

**Graded Problems:**

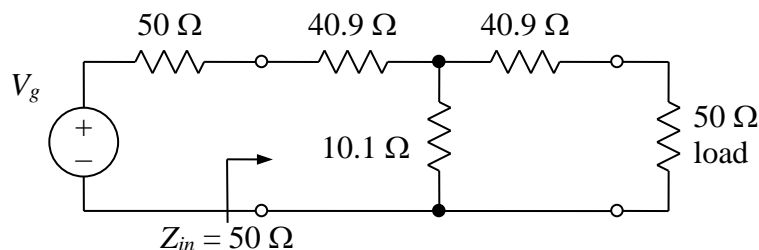
1. Show that the S-parameters  $S_{11}$  and  $S_{21}$  for the network depicted below are expressed in symbolic form by the formulas next to the diagram. The reference characteristic impedance is  $Z_0$ . Assume that the connecting wires have negligible length.



$$S_{11} = \frac{(Z_A - Z_0)(Z_B + Z_C + Z_0) + Z_C(Z_B + Z_0)}{(Z_A + Z_0)(Z_B + Z_C + Z_0) + Z_C(Z_B + Z_0)}$$

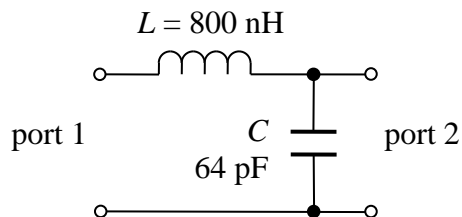
$$S_{21} = \frac{2Z_0Z_C}{(Z_A + Z_0)(Z_B + Z_C + Z_0) + Z_C(Z_B + Z_0)}$$

2. Use the results of the previous problem to find the numerical values of  $S_{11}$  and  $S_{21}$  for the 20 dB T-network attenuator shown below. Express the magnitude of  $S_{21}$  in dB. The reference impedance is  $Z_0 = 50 \Omega$ . The attenuator circuit consists of the  $40.9 \Omega$  and  $10.1 \Omega$  precision resistors shown in the diagram; the other components represent a signal source and a load. Assume that all resistors are ideal. Note that in this case the S-parameters will be real and independent of frequency. Briefly explain the implications of your results; that is, discuss why they make physical sense.

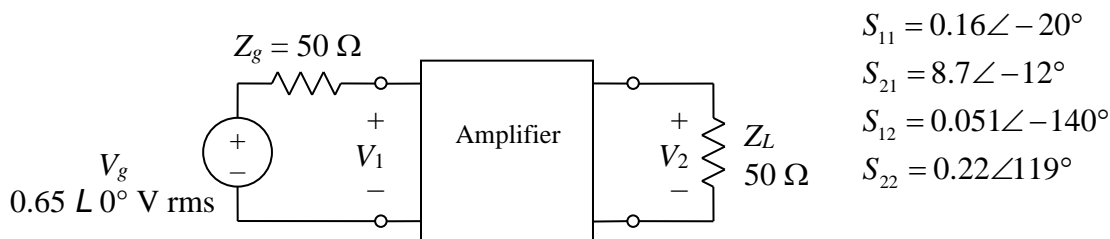


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3. Find the numerical values of all four S-parameters ( $S_{11}$ ,  $S_{21}$ ,  $S_{12}$ , and  $S_{22}$ ) for the L-shaped impedance matching network shown below at an operating frequency of 20 MHz (the design frequency for the matching network). The reference impedance is  $Z_0 = 50 \Omega$ . The network is designed to match the impedance connected to port 2 to a  $50 \Omega$  signal source connected to port 1. Use one or more of the S-parameter values to determine the value of the expected load impedance for which the network was designed.



4. The S-parameters of an amplifier operating at 4.2 GHz are measured and found to have the values listed below the diagram. The reference impedance is  $Z_0 = 50 \Omega$ . As shown in the diagram, a signal source is applied to the input of the amplifier with the indicated phasor source voltage, and a load is connected to the output. Find the power in milliwatts delivered to the load. *Hint*: Find the available power delivered by the source represented by  $V_g$  and  $Z_g$ .

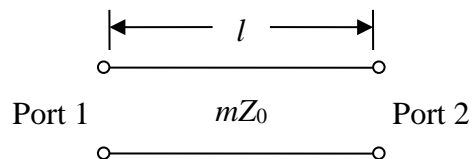


**Ungraded Problems:**

The following problems will not be graded. However, you should attempt to solve them on your own and then check the solutions. Try not to give up too quickly if you struggle to solve them. Move on to a different problem and then come back to the difficult one after a few hours.

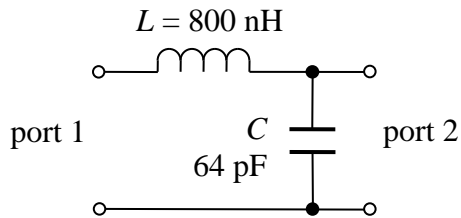
1. Show that the S parameters  $S_{11}$  and  $S_{22}$  of a transmission line of length  $l$  with characteristic impedance  $mZ_0$ , where  $m$  is a real multiplying factor (that's why it's "m") greater than or equal to one, are given by the expression below left. The reference impedance is  $Z_0$ . A diagram of the line section is shown below right. Note that  $S_{11} = S_{22} = 0$  when  $m = 1$ .

$$S_{11} = S_{22} = \frac{j(m^2 - 1) \tan \beta l}{2m + j(m^2 + 1) \tan \beta l}$$



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2. Find the numerical values of all four Z-parameters ( $Z_{11}$ ,  $Z_{21}$ ,  $Z_{12}$ , and  $Z_{22}$ ) for the impedance matching network shown below at an operating frequency of 20 MHz. The network is designed to match the impedance connected to port 2 to a  $50 \Omega$  signal source connected to port 1. Use the conversion formula given below (adapted from D. Pozar, *Microwave Engineering*, 3<sup>rd</sup> ed., John Wiley & Sons, Inc., 2005, Sec. 4.3) to find the S-parameters, and compare their values to the ones found in the corresponding graded problem above. In the conversion formula,  $[Z]$  is the Z-parameter matrix, and  $[Z_0]$  is a diagonal matrix in which all of the entries on the main diagonal are  $Z_0$  ( $50 \Omega$ ).



$$[S] = ([Z] + [Z_0])^{-1} ([Z] - [Z_0])$$

3. Find the S parameters  $S_{11}$  and  $S_{21}$  in symbolic form for an ideal lossless transformer with a turns ratio of  $N:1$  (with  $N$  turns on the port 1 side) as shown below. The reference characteristic impedance is  $Z_0$ . Assume that the connecting wires have negligible length.

