

**Homework Assignment #1 – due via Moodle at 11:59 pm on Monday, Feb. 10, 2025**

***Instructions, notes, and hints***

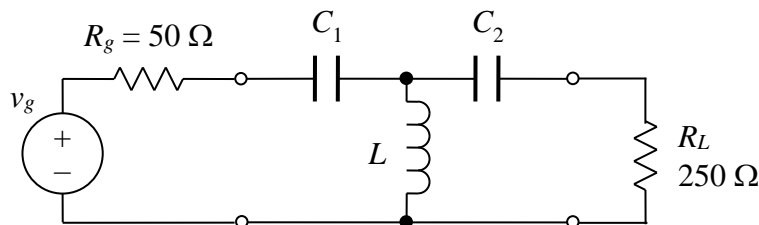
You may make reasonable assumptions and approximations to compensate for missing information, if any. Provide the details of all solutions, including important intermediate steps. You will not receive credit if you do not show your work.

It might be necessary to use reasonable approximations or assumptions to solve some of these problems, especially if critical information is missing. In those cases, your answer might differ from the posted answer by a significant margin. That's okay. If you justify any approximations that you make, you will be given full credit for such answers.

The first set of problems will be graded and the rest will not be graded. Only the graded problems must be submitted by the deadline above. Do not submit the ungraded problems.

***Graded Problems:***

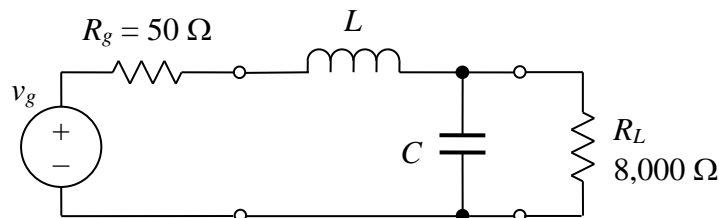
1. An antenna's equivalent impedance of  $250\ \Omega$  needs to be transformed to  $50\ \Omega$  at a frequency of 1.9 MHz, which is the center of the "160-meter" amateur radio band. (The 160-meter band extends from 1.8 to 2.0 MHz. The wavelength at 1.9 MHz is roughly 160 meters.) Design an L network using an inductor as the shunt element to accomplish the impedance transformation. A complete design includes specifications of the capacitor and inductor values using appropriate metric prefixes for the units (e.g., pF instead of F). You do not have to specify standard values.
2. Design a T network with an inductor as the shunt element, if possible, to match a load impedance of  $250\ \Omega$  to  $50\ \Omega$  at a frequency of 1.9 MHz with a target matching bandwidth of 200 kHz. That is, repeat the previous problem but this time with a T network. A diagram of the matching network is shown below. Specify the capacitor and inductor values using appropriate units.



3. Design an L network to match a  $75\ \Omega$  source impedance to a load of  $200 - j150\ \Omega$  at an operating frequency of 50 MHz. Use a capacitor as the shunt (parallel) element, if possible.

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- This problem highlights an issue often encountered when trying to achieve a large impedance transformation ratio with nonideal components. Design an L network to match a load impedance of  $0.5 \Omega$  to  $50 \Omega$  at an operating frequency of 20 MHz. Use a capacitor as the shunt element, and initially assume that the matching components are ideal. After the  $L$  and  $C$  values are obtained, assume that the inductor has a winding resistance of  $R_w = 0.4 \Omega$ , and then calculate the actual transformed impedance obtained with the winding resistance present. *Hint:* The actual impedance differs significantly from the ideal case in which  $R_w = 0$ .
- Another issue commonly encountered in matching networks is power loss. Find the required ideal (no-loss)  $L$  and  $C$  values in the matching network below, assuming that the operating frequency is 14 MHz (low end of the amateur radio “20-meter” band). Now assume that the inductor has a winding resistance of  $R_w = 8.0 \Omega$  at 14 MHz but that all of the other components are still ideal. The input resistance of the network will increase from  $50 \Omega$  to  $58 \Omega$  (Do you know why?), which is an insignificant degradation of the impedance match. Suppose that the signal source is supplying 10 W to the input of the network, which means that the equivalent source voltage  $v_g$  has a magnitude of 44.8 Vrms or 63.4 Vpk. Find the power delivered to the load  $R_L$ , and express the loss from source to load in dB. *Note:* The voltage source  $v_g$  supplies about 20 W to the circuit, but only 10 W is supplied to the input of the network because half of the 20 W is dissipated in  $R_g$ . Remember, though, that power calculations in a Thévenin equivalent circuit (TEC) almost never reflect the actual delivery or absorption of power in the internal circuitry that the TEC represents. You can only draw conclusions from the circuit quantities that you calculate *outside* a TEC.



### Ungraded Problems:

The following problems will not be graded. However, you should attempt to solve them on your own and then check the solutions. Try not to give up too quickly if you struggle to solve any of them. Move on to a different problem and then come back to the difficult one after a few hours.

- Refer to Graded Problems 1 and 2 above. Suppose that a  $50 \Omega$  coaxial cable will connect the signal source to the matching network and load. Write a *Matlab* m-file (script) that plots the VSWR vs. frequency for both networks over the 1–3 MHz range on the same set of axes. Use different line styles (and a legend to identify them) so that the curves can be distinguished in black & white print. The VSWR axis should range from 1 to 6 (the upper limit corresponds a return loss of roughly  $-3$  dB). Add grid lines. Comment on the results.
- The input impedance of a certain small loop antenna can be modeled at its operating frequency as a  $0.4 \Omega$  resistance in series with  $400 \Omega$  of inductive reactance. Use the quality factor ( $Q$ ) method to find the parallel equivalent circuit representation of the antenna’s input impedance. That is, find the values that a resistor and a reactive element ( $L$  or  $C$ ) in parallel would need to have if the combination were to have an equivalent series impedance of  $0.4 + j400 \Omega$ .

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3. Use the results of the previous problem to show that the small loop antenna can be matched to a system impedance of  $50 \Omega$  using a matching network consisting of two capacitors (one in shunt and one in series) added between the antenna and the system. The equivalent parallel inductance of the antenna acts as the third element in the matching network. Find the required capacitor values, and draw a sketch of the matching network and the load, where the latter is represented by its parallel equivalent circuit. Label the capacitors and inductors with their respective reactances. *Hint:* The  $Q$  of the matching network is a large two-digit value.
4. Show that the series equivalent impedance of a resistance  $R_p$  and reactance  $X_p$  in parallel is given by the expression below. Note that this result proves that  $R_s < R_p$  and  $|X_s| < |X_p|$  regardless of the resistance and reactance values in the parallel combination.

$$R_s + jX_s = \frac{R_p}{Q^2 + 1} + j \frac{X_p Q^2}{Q^2 + 1}, \quad \text{where } Q = \frac{R_p}{|X_p|}$$