Homework Assignment #8 – due via Moodle at 11:59 pm on Monday, April 21, 2025

***Instructions, notes, and hints*:**

Provide the details of all solutions, including important intermediate steps. You will not receive credit if you do not show your work.

Use reasonable approximations or assumptions if critical information is missing. In those cases, your answer might differ from the posted answer by a significant margin. That could be okay. If you justify any approximations that you make, you will be given full credit for such answers.

Unless otherwise indicated, you may use Matlab, Mathematica, or other software to make difficult or time-consuming calculations. If you do, include a copy of the file or screen display that shows your work.

The first set of problems will be graded and the rest will not be graded. Only the graded problems must be submitted by the deadline above. Do not submit the ungraded problems.

***Graded Problems*:**

1. The electric field component of a particular uniform plane wave can be described mathematically by the expression given below. The wave is propagating in a "nonmagnetic" medium, which means that $\mu = \mu_o$. The frequency of operation is 50 MHz. Find:

- **a.** a time-domain expression for the electric field in numerical form (i.e., with as many numerical values as possible substituted for variables and parameters).
- **b.** a time-domain expression for the magnetic field **H** in numerical form.
- **c.** the wavelength.
- **d.** the relative permittivity ε_r of the medium.

$$\tilde{\mathbf{E}} = \hat{\mathbf{y}}\,0.2e^{j0.3\pi}e^{-j2.1x} + \hat{\mathbf{z}}\,0.8e^{j0.3\pi}e^{-j2.1x} \ \ \mu\text{V/m}$$

2. A uniform plane wave with a frequency of 1.0 GHz is traveling through Teflon in the $-y$ direction (i.e., $-y$ is the direction of propagation). The electric field vector has a peak value of 0.1 mV/m and points in the direction 45° above the positive x-axis (i.e., bisecting the positive x-axis and the positive z-axis) during half of the wave period and in the opposite direction during the other half. The electric field is at its positive peak at all locations in the y = 2.0 cm plane at time t = 0. Find complete phasor representations of the electric and magnetic fields.

3. A uniform plane wave propagating in the $+z$-direction is partially represented by the mathematical expression shown below. The expression for the y-component (E_y) is missing. Find the mathematical form for E_y that results in the expression below describing a right-hand circularly polarized (RHCP) wave.

$$\tilde{\mathbf{E}} = \hat{\mathbf{x}}\,0.2e^{-j0.1\pi}e^{-j4.8z} + \hat{\mathbf{y}}\,E_y \ \ \text{mV/m}$$

(*continued on next page*)

4. Plot the locus of the electric field vector as a function of time for the waves described by the phasor expressions given below. The "locus of the E-field vector" is the path followed by the tip of the vector (with the start of the arrow at the origin of the coordinate system) as time advances. This is the type of plot that has been produced for the in-class examples. Remember that the E-field vector is simply a mathematical representation. The actual E-field points in the same direction everywhere in any given plane at a particular moment in time. In each case listed below, identify the type of polarization. If it is circular or elliptical, specify the sense (left-hand or right-hand). If linear, specify the angle that the electric field makes with the *y*-axis.

   a. $\tilde{\mathbf{E}} = \hat{\mathbf{y}}\,0.2e^{j0.3\pi}e^{-j2.1x} + \hat{\mathbf{z}}\,0.8e^{j0.3\pi}e^{-j2.1x}$ μV/m
   b. $\tilde{\mathbf{E}} = \hat{\mathbf{x}}\,5.0e^{j0.3\pi}e^{j0.22z} + \hat{\mathbf{y}}\,5.0e^{-j0.7\pi}e^{j0.22z}$ μV/m

5. The electric field component of a particular plane wave can be described mathematically by the expression given below right. Find the magnitude and direction of the electric field in free space at the locations listed below at times $t_1 = 5.0$ ns and $t_2 = 10.0$ ns. Briefly explain the interesting results that you obtain.

   a. $(x, y, z) = (1.0, 1.0, 1.0)$ m
   b. $(x, y, z) = (1.0, 2.0, 3.0)$ m
   c. $(x, y, z) = (1.0, 3.0, 5.0)$ m

   $$\tilde{\mathbf{E}} = \hat{\mathbf{y}}\,0.2e^{j0.3\pi}e^{-j2.1x} + \hat{\mathbf{z}}\,0.8e^{j0.3\pi}e^{-j2.1x} \ \mu\text{V/m}$$

***Ungraded Problems*:**

The following problems will not be graded, but you should attempt to solve them on your own and then check the solutions. Do not give up too quickly if you struggle with one or more of them. Move on to a different problem and then come back to the difficult one after a few hours.

1. A transmitter that is part of a communication link generates a TEM (transverse electromagnetic) plane wave that propagates through air. It can be described mathematically by the expression shown below. The engineers analyzing the communication link define a coordinate system in which the transmitter is located at the origin and the receiving station is located 10 km away along the *x*-axis. Note that the E-field is expressed using spherical coordinates; if you wish, you may convert to Cartesian coordinates in the vicinity of the receiving station.

   a. Determine the polarization of the wave at the receiving site. If the polarization is circular or elliptical, specify the sense (left-hand or right-hand). If linear, specify the angle that the electric field makes relative to the $\hat{\boldsymbol{\theta}}$ direction.
   b. Find the power densities (Poynting vector magnitudes) of the θ and the ϕ-components of the electric field. Also find the power density of the full field.
   c. Find the corresponding phasor expression for the magnetic field in spherical coordinates.

$$\tilde{\mathbf{E}} = \hat{\boldsymbol{\varphi}}\,7.0e^{-j0.2\pi}\frac{e^{-j2.1R}}{R} + \hat{\boldsymbol{\theta}}\,3.8e^{j0.3\pi}\frac{e^{-j2.1R}}{R} \ \text{mV/m}$$

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2. The expression for the electric field in air of an elliptically polarized wave is given below. The frequency of operation is 100 MHz. A center-fed dipole is placed in a plane parallel to the *yz*-plane and is connected to a receiver. Find the orientation with respect to the *y*-axis (i.e., the angle that the dipole makes with the *y*-axis) that the dipole should have to maximize the received signal power P_{rec}, and find the value of P_{rec} for that orientation. Assume that all antennas are 100% efficient and that there are no transmission line or impedance mismatch losses.

$$\tilde{\mathbf{E}} = \hat{\mathbf{y}}5e^{-j0.2\pi}e^{-jkx} + \hat{\mathbf{z}}2e^{j0.3\pi}e^{-jkx} \ \mu\text{V/m}$$

3. Suppose that a transmitting station needs to produce a linearly polarized wave, but it also needs to be able to control the angle of polarization relative to the horizon. One solution is to mechanically rotate a linearly polarized antenna; however, the station is in a cold climate, and ice accumulation could be a problem. An alternative is to mount two antennas with opposite circular polarization (CP) senses (i.e., one RHCP and the other LHCP) side-by-side and introduce a phase shift between them. There are circuits that can shift phase, and one could be inserted at the input of one of the CP antennas. The signals radiated by the two antennas can be represented by the mathematical expressions shown below, where A is a real constant, and $k = 2\pi/\lambda$. The distance from the pair of antennas to the receiving site is far enough that the radiated fields can be considered plane waves. The subscripts L and R indicate the polarization sense (left-hand and right-hand, of course). Use the superposition principle to show symbolically that the two signals add to produce a linearly polarized wave, and determine how the phase shift $\Delta\phi$ is related to the angle ζ that the E-field makes with the horizon. (The *x*-axis is parallel to the horizon.). A potentially helpful identity is included below.

$$\tilde{\mathbf{E}}_L = \hat{\mathbf{x}}\, Ae^{-jkz} + \hat{\mathbf{y}}\, Ae^{j0.5\pi}e^{-jkz} \quad \text{and} \quad \tilde{\mathbf{E}}_R = \hat{\mathbf{x}}\, Ae^{j\Delta\phi}e^{-jkz} + \hat{\mathbf{y}}\, Ae^{-j0.5\pi}e^{j\Delta\phi}e^{-jkz}$$

$$1 + e^{j\Delta\phi} = e^{j0.5\Delta\phi}\left(e^{-j0.5\Delta\phi} + e^{j0.5\Delta\phi}\right) = e^{j0.5\Delta\phi}\left(e^{j0.5\Delta\phi} + e^{-j0.5\Delta\phi}\right) = 2e^{j0.5\Delta\phi}\cos\left(0.5\Delta\phi\right)$$