ECEG 390 Theory and Applications of Electromagnetics Spring 2025

Homework Assignment #7 – due via Moodle at 11:59 pm on Tuesday, Apr. 15, 2025

***Instructions, notes, and hints*:**

Provide the details of all solutions, including important intermediate steps. You will not receive credit if you do not show your work.

Use reasonable approximations or assumptions if critical information is missing. In those cases, your answer might differ from the posted answer by a significant margin. That could be okay. If you justify any approximations that you make, you will be given full credit for such answers.

Note that the first set of problems will be graded and the rest will not be graded. Only the graded problems must be submitted by the deadline above. Do not submit the ungraded problems.

***Graded Problems*:**

1. A highly advanced circular reflector antenna has been fabricated using a new conductive polymer. The physical aperture is 67 cm in diameter, but because of "spillover" from the feed antenna, the aperture efficiency is only 60%. Although the antenna presents a 50 Ω load to the 50 Ω transmission line feeding it, measurements indicate that 10 Ω of the 50 Ω total is due to power loss from the limited conductivity of the polymer, the type of feed arrangement used, and various losses in the feed antenna. (Note that aperture efficiency and power efficiency are different effects and can exist simultaneously.) The antenna is to be used in an 8.0 GHz communication link between two identical facilities that are 20 km apart. Each one has one of the new antennas and a transmitter that produces 5.0 W of output power; however, the transmission lines that connect the transmitters and receivers to the antennas each have 6.0 dB of loss. (The antenna can be switched between transmission and reception mode.) The antennas are perfectly aligned toward each another so that each one is in the other's direction of maximum radiation.

- **a.** Find the power density in nW/m^2 in the vicinity of the antenna at one of the facilities when the other is transmitting. Neglect atmospheric attenuation and reflections from the ground and other objects between the facilities.
- **b.** Find the signal power that appears at the input of the receiver when one of the facilities is being used in reception mode.

2. A satellite ground station consists of a 1.0 kW transmitter and an antenna with a directivity of 26 dBi and an efficiency of 50%. The satellite's receive antenna has a directivity of 20 dBi and an efficiency of 90%. Find the variation in the signal power detected by the satellite's receiver as its orbit causes its distance to the ground station to vary from 200 km to 3000 km. Neglect losses in the atmosphere, losses in the interconnecting transmission lines, and reflections from the ground, and assume that the ground station antenna tracks the satellite so that it is always in the direction of maximum gain. For free space, $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m, $\mu_0 = 4\pi \times 10^{-7}$ H/m, and $\eta = 120\pi\,\Omega$ (or 377 Ω). The frequency of operation is 12 GHz.

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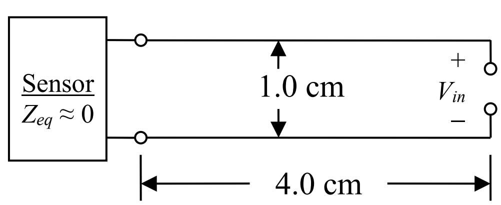
3. In a semi-rural area like Lewisburg, the noise levels in the AM broadcast band are such that a receiver detects roughly 0.5 nW of noise when tuned to a particular station. An incoming AM signal must be at least 10 dB above the noise level to be considered listenable by most people. The WMLK-AM transmitter operates at a frequency of 1070 kHz and is approximately 10 km from Lewisburg. It uses a monopole antenna with a gain of about 5 dBi. A receiver that is tuned to the signal has a 1.0 m-long center-fed short dipole antenna (triangular current distribution) made from an aluminum rod with a diameter of 4.8 mm. Find the minimum transmitter power required to maintain a listenable signal at the receiver. Assume a direct path between the receiving antenna and the transmitting antenna, neglect reflections from the ground and nearby objects, and assume that the transmission line and impedance matching network losses are negligible.

4. The receiver of a garage door opener (the part inside the garage) uses a circular loop antenna with a diameter of 2.0 cm. The opener operates at a frequency of 915 MHz. At the input terminals of the loop, a voltage with a peak value of at least 100 μV must be developed if the door is to open.

- **a.** If the loop is oriented for maximum response (i.e., maximum voltage across the terminals), find the peak magnitude of the magnetic field **H** in the vicinity of the loop.
- **b.** The loop is now tilted 60° away from the position of maximum response. For the same magnetic field as in part **a**, find the voltage that would be developed across the gap with the new orientation.

5. An engineering student builds a sensitive heart rate detector circuit on a piece of single-sided printed circuit board. (That is, there are copper traces on one side and no copper anywhere on the other side.) The copper circuit board traces from the sensor leading to the first stage of amplification are parallel, 4.0 cm long, and spaced 1.0 cm apart. The sensor, the copper traces, and the input terminals of the amplifier form a circuit loop. The sensor system is depicted in the diagram below. A nearby FM radio station operating at 100 MHz generates an electric field **E** that has a peak magnitude of 40 mV/m in the vicinity of the detector. Find the peak voltage at 100 MHz that is developed across the input of the amplifier by the FM signal when the PC board is oriented for maximum response to the signal. Assume that the sensor has an equivalent impedance of nearly zero at 100 MHz. Recall that the electric field and magnetic field magnitudes of the transverse electromagnetic (TEM) fields radiated by all antennas have the relationship $|\mathbf{E}|/|\mathbf{H}| = \eta$, where η is the intrinsic impedance of the medium in which the wave propagates. For air and free space, $\eta = 120\pi\,\Omega = 377\ \Omega$.



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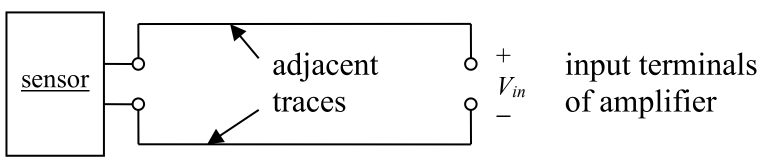
6. The engineering student in the previous problem realizes that the FM radio station is being picked up by the heart rate detector's input circuitry and is interfering with its proper operation. The student redesigns the PC board by reducing the separation between the traces coming from the sensor from 1.0 cm to 1.0 mm. Find the largest possible peak voltage at 100 MHz that will be developed across the input of the amplifier with the new design. Assume that the width of the circuit board traces is negligible compared to the spacing between the traces.

***Ungraded Problem*:**

The following problem will not be graded, but you should attempt to solve it on your own and then check the solution. Do not give up too quickly if you struggle with it. Move on to something else and then come back to it after a few hours.

1. The interference from the FM station considered in the last two graded problems can be reduced even further by using a twisted pair of wires from the sensor to the amplifier's input terminals. A comparison of the two feed arrangements is shown below. In the upper diagram, the leads from the sensor to the amplifier are straight and parallel. In the lower diagram, the leads are twisted. (You may assume that the wires in the twisted pair form either flat loops or helices; the answer is the same either way.) Provide a brief qualitative explanation based on Faraday's law for why a twisted pair usually has better signal rejection than two parallel wires; that is, explain why the voltage induced at the terminals when a twisted pair is used is much less than that induced when parallel wires are used.



H-field vector points into/out of page.