ECEG 390 Theory and Applications of Electromagnetics Spring 2025

Homework Assignment #6 – due via Moodle at 11:59 pm on Monday, Mar. 31, 2025
[revised 3/28/2025; Graded Probs. 4–6 deferred to HW #7]

***Instructions, notes, and hints*:**

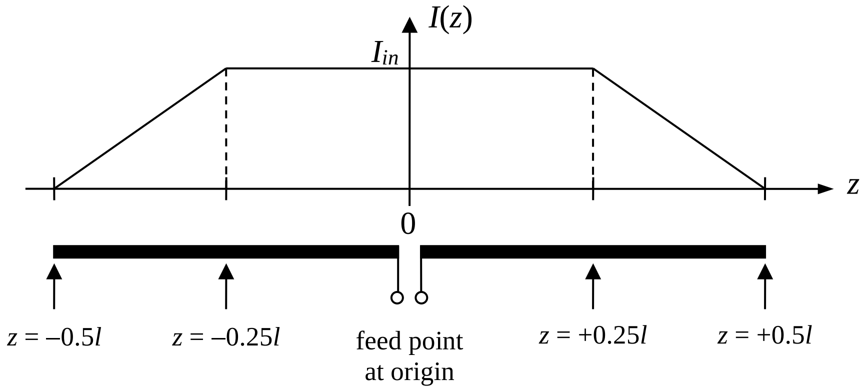
Provide the details of all solutions, including important intermediate steps. You will not receive credit if you do not show your work.

It might be necessary to use reasonable approximations or assumptions to solve some of these problems, especially if critical information is missing. In those cases, your answer might differ from the posted answer by a significant margin. That could be okay. If you justify any approximations that you make, you will be given full credit for such answers.

Note that the first set of problems will be graded and the rest will not be graded. Only the graded problems must be submitted by the deadline above. Do not submit the ungraded problems.

***Graded Problems*:**

1. An electrically short ($l << \lambda$) center-fed dipole antenna operating at 10 MHz has an unusual design feature that causes the current distribution along its length to have the functional dependence shown below. The input current at the feed point is I_{in}. The current magnitude is constant over the middle half of the antenna, and it tapers linearly to zero at the ends over the outer sections. Derive an expression for the far electric field radiated by the antenna, and then determine the directivity of the antenna. The current does not experience a phase shift along the antenna because it is a standing wave; therefore, I_{in} can be assumed to have a real value, and $I(z)$ has a constant phase of 0° along the antenna. (That is, the current is a real-valued function of z.) Assume that the tiny gap at the feed point infinitesimally narrow. The antenna is made from aluminum rod stock with a diameter of 3.18 mm (about 1/8 in).



2. For the antenna in the previous problem, derive an expression for the radiation resistance as a function of the length l. Obtain a numerical value for $l = 0.02\lambda$, and compare it to the radiation resistance of a standard short dipole (with a triangular current distribution) and that of a Hertzian dipole (uniform distribution) of the same length.

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3. For the antenna in the previous two problems, derive an expression for loss resistance R_{loss} as a function of the length l. Obtain numerical values for R_{loss} and the efficiency for $l = 0.02\lambda$, and compare them to those of a short dipole (triangular current distribution) and a Hertzian dipole (uniform distribution) of the same length. The input current at the feed point is I_{in}.

4. **[deferred to HW #7]** A highly advanced reflector antenna has been built out of a new conductive polymer. The physical aperture of the antenna is 67 cm in diameter, but because of "spillover" from the feed antenna, the aperture efficiency is only 60%. Although the antenna presents a 50 Ω load to the 50 Ω transmission line feeding it, measurements indicate that 10 Ω of the 50 Ω total is due to power loss from the limited conductivity of the polymer, the type of feed arrangement used, and various losses in the feed antenna. (Note that aperture efficiency and power efficiency are different effects and can exist simultaneously.) The antenna is to be used in an 8.0 GHz communication link between two facilities that are 20 km apart. The facilities are identical, and each one has one of the new antennas and a transmitter that can produce 5.0 W of output power; however, the 40 m long transmission line that connects the transmitters and receivers to each antenna each introduce 6.0 dB of loss. (The antenna can be switched between transmission and reception mode.) The antennas are perfectly aligned toward one another so that each one is in the other's direction of maximum radiation.

   a. Find the power density in nW/m^2 in the vicinity of the antenna at one of the facilities when the other is sending a signal. Neglect atmospheric attenuation and reflections from the ground and other objects between the facilities.
   b. Find the signal power that appears at the input of the receiver when one of the facilities is being used in reception mode.

5. **[deferred to HW #7]** A satellite ground station consists of a 1.0 kW transmitter and an antenna with a directivity of 26 dBi and an efficiency of 50%. The satellite's receive antenna has a directivity of 20 dBi and an efficiency of 90%. Find the variation in the signal power detected by the satellite's receiver as its orbit causes its distance to the ground station to vary from 200 km to 3000 km. Neglect losses in the atmosphere, losses in the interconnecting transmission lines, and reflections from the ground, and assume that the ground station antenna tracks the satellite so that it is always in the direction of maximum gain. For free space, $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m, $\mu_0 = 4\pi \times 10^{-7}$ H/m, and $\eta = 120\pi\,\Omega$ (or 377 Ω). The frequency of operation is 12 GHz.

6. **[deferred to HW #7]** In a semi-rural area like Lewisburg, the noise levels in the AM broadcast band are such that a receiver detects roughly 0.5 nW of noise when tuned to a particular station. An incoming AM signal must be at least 10 dB above the noise level to be considered listenable by most people. An AM transmitter operates at a frequency of 1070 kHz and is approximately 10 km from Lewisburg. It uses a monopole antenna with a gain of about 5 dBi. A receiver that is tuned to the signal has a 1 m-long center-fed short dipole antenna made from an aluminum rod with a diameter of 4.8 mm. Find the minimum transmitter power required to maintain a listenable signal at the receiver. Assume a direct path between the receiving antenna and the transmitting antenna, neglect reflections from the ground and nearby objects, and assume that the transmission line and impedance matching network losses are negligible.

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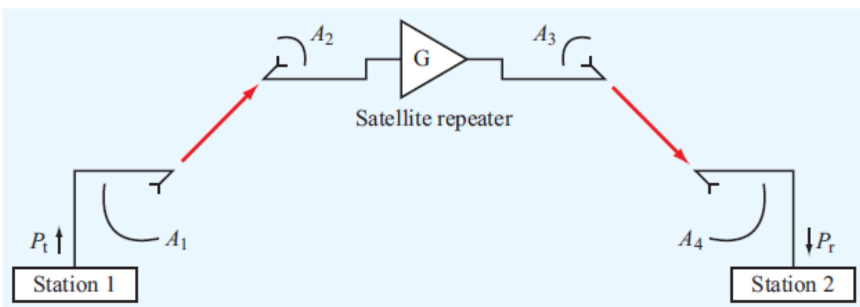
***Ungraded Problems*:**

The following problems will not be graded, but you should attempt to solve them on your own and then check the solutions. Do not give up too quickly if you struggle with one or more of them. Move on to a different problem and then come back to the difficult one after a few hours.

1. Find the effective aperture in terms of square wavelengths (λ^2) for the following antennas. Also, find the radius in terms of λ of the circular area that is equivalent to the effective aperture, and compare the radius to the size of the antenna.

   a. standard center-fed half-wave dipole
   b. center-fed dipole of length 0.1λ
   c. Yagi-Uda array with a total length of 3.83λ, and a directivity of 15.8 dBi

2. [adapted from Prob. 9.29 of the textbook (Ulaby and Ravaoli, 7th ed.)] The figure below depicts a satellite repeater with two antennas, one pointed towards ground station 1 and the other towards ground station 2. All antennas are parabolic dishes, each with an aperture efficiency of 60%. The transmission line between each ground station and its respective antenna has a loss of 3 dB. The line loss between the satellite antennas and the repeater is negligibly small. Antennas A_1 and A_4 are each 4.0 m in diameter, and antennas A_2 and A_3 are each 2.0 m in diameter. The distance between the satellite and each of the ground stations is 40,000 km. The satellite transponder receives the signal from station 1 using antenna A_2, amplifies the signal by 80 dB, and then uses antenna A_3 to retransmit the signal to ground station 2. The system operates at 10 GHz with P_t = 1.0 kW (transmitter power at station 1). Determine the received power P_r at station 2.



[Figure source: F. T. Ulaby and U. Ravaioli, Fundamentals of Applied Electromagnetics, 7th ed., Pearson Education, Inc., Upper Saddle River, NJ, 2025.]

3. The WVBU transmitter on the roof of the Dana Engineering Building has an output power of around 400 W, which is supplied to an antenna with a gain of around 6.0 dBi. Assume that the transmission line between the transmitter and the antenna has 2.0 dB of loss and that all receivers are in the direction of maximum radiation from the transmit antenna. Suppose that a receiver tuned to WVBU at 90.5 MHz requires a received signal strength of –100 dBm (10^{-13} W) to produce mostly noise-free sound. The receiver uses a half-wave dipole antenna oriented so that the signal arrives from a direction that is within the half-power beamwidth of the antenna. Find the maximum distance between the receiver and the WVBU transmitting site that allows mostly noise-free reception. Assume a direct line-of-sight path between the transmit and receive antennas, and neglect reflections from the ground and nearby objects.

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4. Horn antennas are analogous to the bells on brass and woodwind musical instruments, and they are often used in antenna measurement facilities since they are structurally rugged, relatively easy to use, and maintain their calibration very well. One type of horn antenna is made from a flared waveguide that ends in a rectangular opening (aperture). With a carefully calibrated gain specification, it can be used as a gain standard in antenna test ranges. The commercially available antenna depicted below (diagram taken from the datasheet) has a 42 mm × 37 mm aperture and a gain of 10.9 dBi at 10.0 GHz. Assuming that the power efficiency of the antenna is 90%, find its aperture efficiency.