ECEG 390 Theory and Applications of Electromagnetics Spring 2025

Homework Assignment #3 – due via Moodle at 11:59 pm on Tuesday, Feb. 25, 2025

***Instructions, notes, and hints*:**

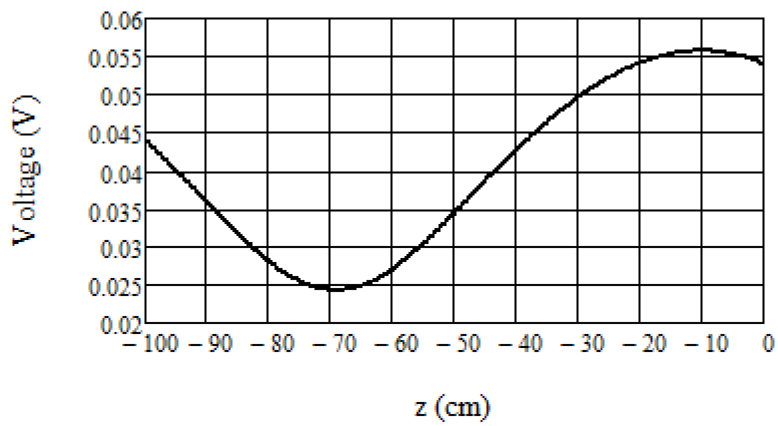
You may make reasonable assumptions and approximations to compensate for missing information, if any. Provide the details of all solutions, including important intermediate steps. You will not receive credit if you do not show your work.

It might be necessary to use reasonable approximations or assumptions to solve some of these problems, especially if critical information is missing. In those cases, your answer might differ from the posted answer by a significant margin. That could be okay. If you justify any approximations that you make, you will be given full credit for such answers.

Note that the first set of problems will be graded and the rest will not be graded. Only the graded problems must be submitted by the deadline above. Do not submit the ungraded problems.

***Graded Problems*:**

1. Suppose that you have just formed a company that will market antennas designed for a new point-to-point wireless broadband delivery service. You cannot afford an expensive vector network analyzer, so instead you purchase an old slotted line to make impedance measurements. The slotted line is essentially lossless, has air insulation, and has a characteristic impedance of 50 Ω. You connect the line to your latest prototype antenna and obtain the plot shown below of the voltage magnitude vs. distance from the load. Find the antenna's input impedance.



2. Find the phasor representation (in polar form) of the load voltage V_L at the location of the open circuit at the far end of the transmission line stub shown below. Also find the phasor input current I_{in} of the stub. Express both quantities in polar form. The operating frequency is 10 MHz, and the line is a parallel-wire type with air insulation.

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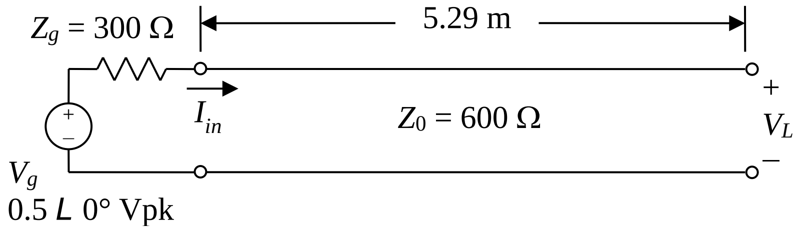
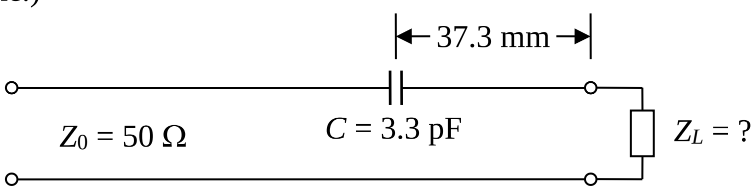


Diagram for Graded Prob. 2

3. A 50 Ω microstrip line will be used to supply a wireless broadband signal to an amplifier with an input impedance of 30 + j45 Ω and that operates at a frequency of 2.45 GHz. The effective relative permittivity of the microstrip line is 4.2. Find the closest location to the load (i.e., the point nearest the amplifier's input terminals) at which an SMT (surface mount technology) capacitor can be inserted in series with the line to achieve an impedance match. Also find the required capacitance value in picofarads.

4. Attempt to design a series-element matching system to achieve an impedance match between a load of $Z_L = j40\ \Omega$ and a transmission line with $Z_0 = 75\ \Omega$. The dimension of the line section may be in wavelengths. Explain conceptually why this task is impossible.

5. Suppose that you are working for a company that has found an old piece of equipment with the microstrip matching system depicted schematically below. The load is an antenna, but there is no documentation that records its input impedance. However, you can measure the dimensions of the microstrip line, and you can read the value of the surface mount capacitor C. You also know that the system operated at 1.6 GHz and that the microstrip line has an effective relative permittivity of $\varepsilon_r = 4.0$. Use the given information to estimate the input impedance of the antenna. (The antenna's input impedance is the load impedance of the transmission line.)



6. An antenna with an equivalent input impedance of 250 + j150 Ω is to be connected to a parallel-wire air-insulated transmission line with $Z_0 = 450\ \Omega$. The operating frequency is 20 MHz. The designers would like to achieve an impedance match by inserting an inductor or capacitor in parallel with the line at an appropriate distance from the load. Determine the location nearest to the load at which each type of component should be inserted. Also specify the required inductance and capacitance values.

7. Suppose in the previous problem that a shorted stub will be used at each location instead of a lumped reactance. Find the required physical length of each stub. The stubs are to be made from sections of 450 Ω parallel-wire transmission line with air insulation.

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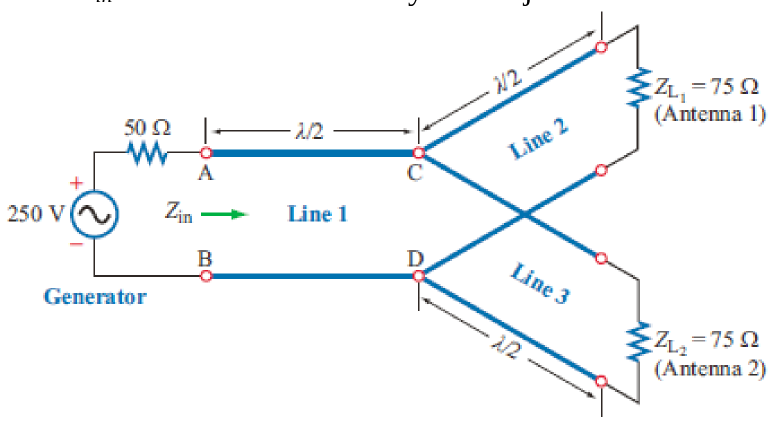
***Ungraded Problems*:**

The following problems will not be graded, but you should attempt to solve them on your own and then check the solutions. Do not give up too quickly if you struggle with one or more of them. Move on to a different problem and then come back to the difficult one after a few hours.

1. Use the expression given below for the input impedance of a loaded transmission line to show that any line with a purely reactive load (i.e., $Z_L = jX$, where X can be a capacitive or inductive reactance) must have an input impedance of zero at those locations $z_{\min}$ where voltage minima occur. You may use one of the expressions for $d_{\min}$ given in the textbook.

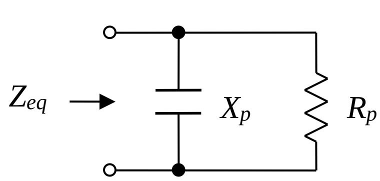
$$Z_{in} = Z_0 \frac{1+\Gamma e^{j2\beta z}}{1-\Gamma e^{j2\beta z}} = Z_0 \frac{1+|\Gamma| e^{j(\theta_r+2\beta z)}}{1-|\Gamma| e^{j(\theta_r+2\beta z)}}$$

2. As shown below, two antennas are being fed by a single signal source. Line 1 has a characteristic impedance of 50 Ω, and lines 2 and 3 each have characteristic impedances of 75 Ω. Find the VSWR along each of the line sections (i.e., along lines 1, 2, and 3). Also find the input impedance Z_{in} of the whole antenna system at junction A-B.



Source: F. T. Ulaby and U. Ravaioli, *Fundamentals of Applied Electromagnetics*, 7th ed., Pearson Education, Inc., Upper Saddle River, NJ, 2015.

3. The formula for l_{main} shown below is used in the design of shunt element-based matching networks. Suppose that a network is to be designed to match a load impedance of 19.5 – j24.4 Ω to a 75-Ω transmission line. At the distance l_{main} from the load, the equivalent input impedance can be modeled by the parallel equivalent circuit shown below (represented by R_p and X_p; the p subscripts are for "parallel"). Find the values of R_p and X_p that constitute an appropriate model for this case. Note that Z_{eq} is the impedance seen looking into the line toward the load at the distance l_{main} from the load. The dimensions of the equivalent circuit below are tiny compared to a wavelength.



$$l_{main} = \frac{\lambda}{4\pi}\left[\theta_r \pm \cos^{-1}\left(-|\Gamma|\right)\right]$$

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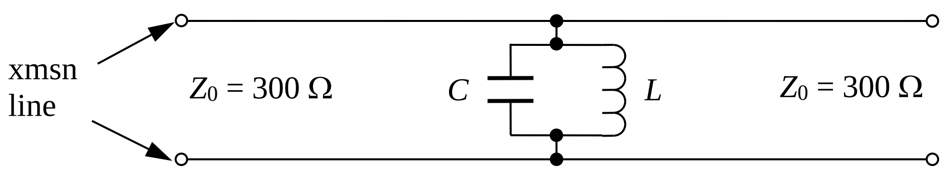
4. Imagine a capacitor and an inductor connected across a transmission line at the same point, as shown below. The components have reactances of equal magnitude but opposite algebraic sign at the operating frequency, so they form a parallel resonant circuit that has an equivalent combined impedance of infinity. That means that the presence of the circuit is not "felt" by waves propagating along the line at the resonant frequency. However, waves at other frequencies will experience partial reflection at the location of the LC circuit, with greater reflection at frequencies farther from resonance. The line and LC circuit therefore form a type of band-pass filter. Although lumped capacitors and inductors can be used, more typically transmission line stubs are employed. Find the electrical lengths (i.e., in wavelengths) of a pair of shunt (parallel) stubs necessary to create reactances of

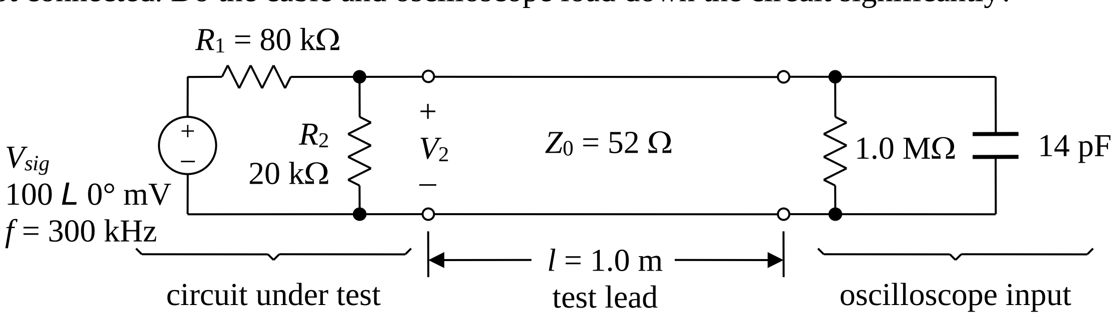
a. $X_L = |X_C| = 30\ \Omega$
b. $X_L = |X_C| = 3000\ \Omega$,

where X_L is the equivalent inductive input reactance of one stub, and X_C is the equivalent capacitive input reactance of the other stub. Use open-circuited stubs. Both stubs are connected in parallel with the line at the same point (where the inductor and capacitor are connected in the diagram below).

For each case, add the lengths of the two stubs together. Do you notice something interesting? What would the stub structure look like if $X_L = |X_C| \to \infty$?



5. Suppose that you have been asked to measure the voltage across a voltage divider that operates at 300 kHz. The figure below depicts the signal source V_{sig} and the divider formed by R_1 and R_2. The circuit is electrically small (i.e., it is less than 0.01λ in size). To make the measurements, you are using an oscilloscope with a test lead made from a 1.0 m length of RG-58A coaxial cable ($Z_0 = 52\ \Omega$ with polyethylene dielectric). As shown in the figure, the oscilloscope's input port can be modeled as a resistance of 1.0 MΩ in a parallel with a capacitance of 14 pF. (These values are printed next to the input ports of most oscilloscopes, including the ones in the Bucknell ECE labs.) Find the phasor voltage V_2 (in polar form) that would be measured across the 20 kΩ resistor with the test lead connected across the resistor as shown. Compare your answer to the voltage that would appear across R_2 if the leads were not connected. Do the cable and oscilloscope load down the circuit significantly?



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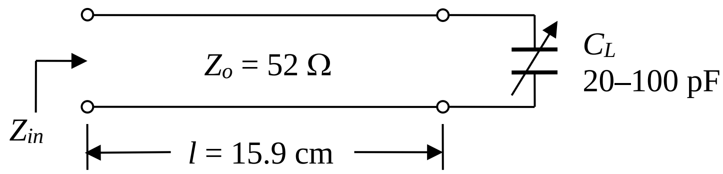
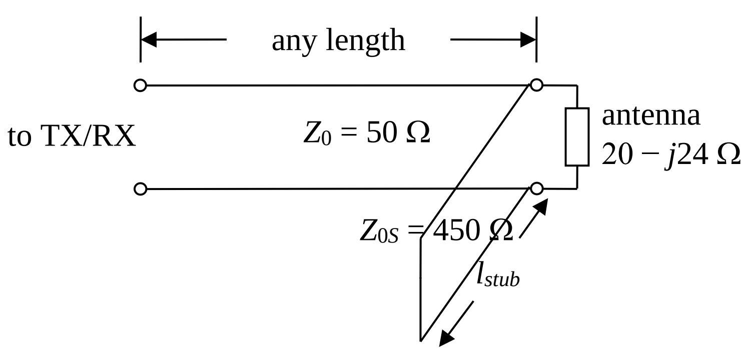
6. As shown below, a 20–100 pF variable capacitor is attached to the end of a 15.9 cm long section of RG-58A coaxial transmission line ($Z_0 = 52\ \Omega$ with polyethylene insulation). The operating frequency is 90.5 MHz, which is the assigned frequency for Bucknell's FM radio station WVBU.

- **a.** Find the input impedance Z_{in} of the line for the minimum value of C_L (20 pF).
- **b.** Find the input impedance when C_L is at the high end of its range (100 pF).
- **c.** If possible, find the value of C_L (in pF) at which the line's input impedance Z_{in} is zero at 90.5 MHz.
- **d.** For the capacitance found in part c, find the input impedance of the line at 10 kHz.

The loaded stub considered in this problem is sometimes called a "wave trap." It is an effective way to keep a troublesome signal at a specific high frequency out of test equipment while minimizing the effect on low-frequency signals. The wave trap shown below would be placed in parallel with the test leads at the input port of a piece of test equipment (like an oscilloscope).



7. A class of antennas known as Yagi-Uda arrays (after the Japanese engineers who invented the concept in the 1920s) typically have low input impedances relative to commonly used 50 Ω coaxial cable. One way to achieve an impedance match is to adjust one of the parts of the antenna so that the input impedance is complex with a negative imaginary part (capacitive). Then an inductive stub is connected in parallel with the antenna's terminals, and the result is an excellent match. Depicted below schematically is the feedline and input impedance of a Yagi-Uda array that has already been adjusted to have an appropriate capacitive input impedance. Find the required length of a shorted stub that would result in a good match to 50 Ω. Note that the characteristic impedance Z_{0S} ("*S*" for stub) of the line section that makes up the stub is 450 Ω, not 50 Ω; it is made from a short piece of parallel-wire line rather than coax. The frequency of operation is 144 MHz (the amateur radio 2-meter band). This type of impedance matching system is called a "hairpin" match; many examples can be found online by searching the term "hairpin match yagi."



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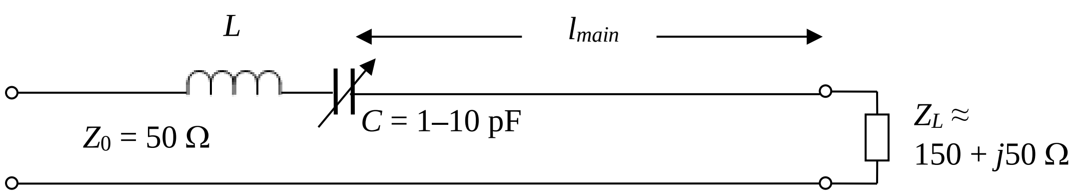
8. A load of roughly $150 + j50\ \Omega$ at an operating frequency of 1.6 GHz is connected to a 50 Ω microstrip line with $\varepsilon_r = 4.0$. Because of manufacturing issues, the load is likely to vary considerably from the indicated value. An important goal is to make all parts of the system as small as possible. The experienced designers know that the closest point to an inductive load where a series matching element can be placed is likely to have a negative input reactance (X_{in}) and therefore would require an inductor to achieve a match. They would like to use a variable inductor to accommodate the variable load impedance, but inductors are bulky, expensive, and often lossy in the RF range. The designers can implement a variable inductance using a fixed inductor in series with a variable capacitor as shown below, but the only available variable capacitor range is 1–10 pF. (The drawing is not to scale. The size of the LC combination is much less than l_{main}.) Find the required fixed inductance value L so that the range of adjustability of the LC combination is centered on the required inductive reactance value. Also find the required distance l_{main} from the load in millimeters.



9. A day-care center wants to install a closed-circuit television (CCTV) system to monitor the facility using old donated TV sets, which will all be tuned to old NTSC channel 3 (center frequency of 69 MHz). Rough measurements reveal that the TV sets have input impedances of roughly $290 + j50\ \Omega$. The CCTV distribution system uses 75 Ω coaxial cable with polyethylene insulation, so a single shunt-stub matching system made from the same cable will be added to each TV set. A "T" connector will be used to connect the stub in parallel with the main transmission line. To allow for the unknown input impedances of the TV sets, the length of each stub will be either $\lambda/4$ or $\lambda/2$, and a variable capacitor will be mounted at the end of each stub to provide adjustability. The matching system is depicted below.

- **a.** Find the shortest distance d from the load at which the stub should be placed.
- **b.** Given that a variable capacitor terminates each stub, determine whether its length should be $\lambda/4$ or $\lambda/2$. Think about how the capacitive load impedance would be transformed in each case. Find the physical length of the selected stub in cm.
- **c.** Find the center (average) value of the variable capacitor. Variable capacitors have maximum and minimum values, of course, so an actual system would be designed so that the average expected required capacitance would fall in the middle of the range. That should allow enough adjustability to achieve a good – but maybe not perfect – impedance match regardless of the actual load impedance.