

Homework Assignment #3 – due via Moodle at 11:59 pm on Monday, Sept. 23, 2024

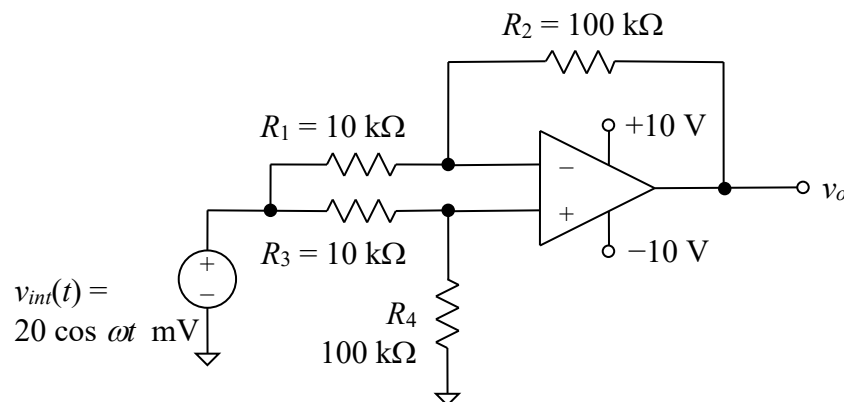
Instructions, notes, and hints:

You may make reasonable assumptions and approximations in order to compensate for missing information, if any. Provide the details of all solutions, including important intermediate steps. You will not receive credit if you do not show your work.

The first few problems will be graded and the rest will not be graded. Only the graded problems must be submitted by the deadline above. Do not submit the ungraded problems.

Graded Problems:

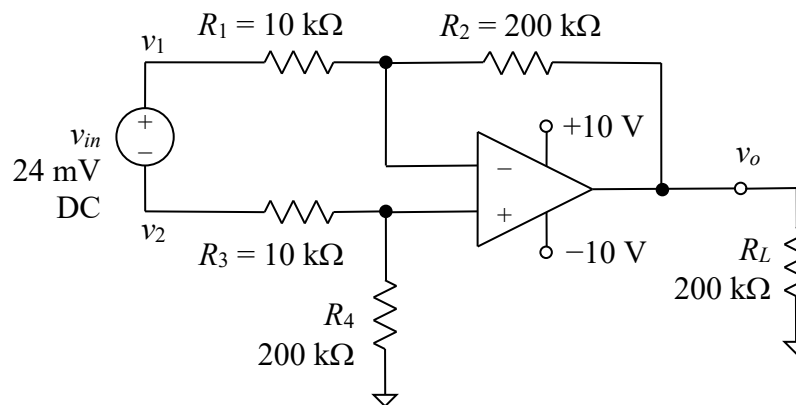
1. The circuit shown below models a diff amp that has its input terminals shorted together so that the differential-mode input voltage is zero. However, the diff amp does experience a common-mode interference signal $v_{int}(t)$ at its inputs. The interference frequency is $f = 100$ kHz. Nominal resistor values are shown, but the resistor tolerance is 5%. Assuming that the actual resistor values are no more than $\pm 5\%$ away from their respective nominal values, find the worst-case magnitude of the common-mode output voltage v_o .



2. Suppose that you have been asked to design a new digital data link, and the system needs a basic diff amp like the one shown in Figure 2.16 in the textbook (Sedra & Smith, 8th ed.). The differential-mode gain must be around 20 V/V, and the differential-mode input resistance must be $R_{id} = 50$ k Ω . The power supply voltages are to be ± 10 V. The selected op-amp has a maximum rated output current of 30 mA, a maximum input bias current of 20 nA, and a peak input offset voltage magnitude of 1.5 mV. Find the maximum allowable tolerance for the resistors R_1 through R_4 to ensure that the CMRR of the diff amp would be at least 75 dB.

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3. The resistors in the diff amp depicted below are labeled with their nominal (ideal) values. Each one has a 2% tolerance. The actual resistor values (to four-digit accuracy) are $R_1 = 10.10 \text{ k}\Omega$, $R_2 = 196.4 \text{ k}\Omega$, $R_3 = 10.14 \text{ k}\Omega$, and $R_4 = 198.8 \text{ k}\Omega$. A sensor is generating a quasi-DC (i.e., very slowly varying) input voltage that currently has a value of 24 mV. The sensor is connected to the amplifier via a long cable in a noisy environment. The building's AC wiring causes a common-mode input voltage expressed as $v_{Icm}(t) = 0.140 \cos(377t) \text{ V}$ to appear at each input of the amplifier. Note that $|v_{in}| < |v_{Icm}|_{pk}$. Find:
- the nominal differential-mode gain (i.e., the gain assuming ideal resistor values).
 - the actual differential-mode gain A_d (using actual resistor values).
 - the actual common-mode gain A_{cm} .
 - the actual CMRR (in dB).
 - the differential-mode output voltage v_{od} .
 - the common-mode output voltage v_{ocm} .



4. Suppose that the resistors in the diff amp in the previous problem are replaced with new ones that have 5% tolerance. The actual resistor values are now $R_1 = 10.42 \text{ k}\Omega$, $R_2 = 191.0 \text{ k}\Omega$, $R_3 = 10.38 \text{ k}\Omega$, and $R_4 = 205.6 \text{ k}\Omega$. Repeat parts **a** through **f** of the previous problem. Comment on the relative changes in the differential-mode gain and the common-mode gain; that is, how do the magnitudes of the changes compare to each other, and why?
5. In the supplemental reading “Real-World Performance of Difference Amplifiers,” it is shown that the worst-case CMRR for a diff amp occurs when

$$R_1 = (1 - \varepsilon)R_{1nom} \quad R_2 = (1 + \varepsilon)R_{2nom} \quad R_3 = (1 + \varepsilon)R_{3nom} \quad R_4 = (1 - \varepsilon)R_{4nom},$$

where the “nom” subscript indicates the nominal (ideal) value of the resistor and where ε is the fractional tolerance (e.g., $\varepsilon = 0.05$ corresponds to 5%). The same worst-case CMRR is found if the “+” signs are replaced with “-” signs and vice versa above. Using the worst-case resistor values given above, show that the approximation given below left is valid regardless of the nominal resistor values. In a diff amp, the nominal resistor values have the relationship given below right.

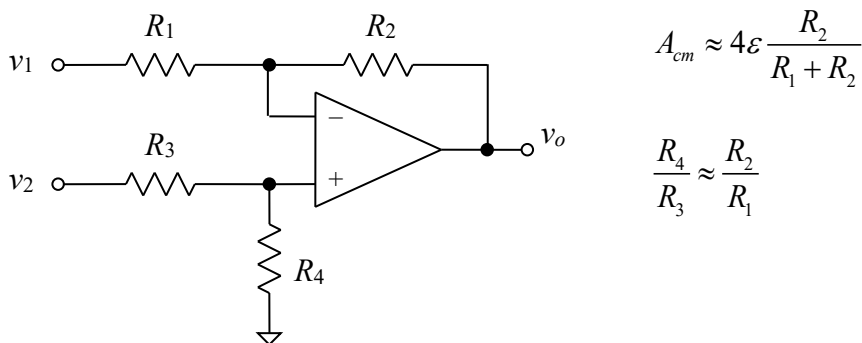
$$1 - \frac{R_2}{R_1} \frac{R_3}{R_4} \approx 4\varepsilon \qquad \frac{R_{2nom}}{R_{1nom}} \frac{R_{3nom}}{R_{4nom}} = 1$$

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Ungraded Problems:

The following problems will not be graded. They are intended to serve as practice problems and examples. You should attempt to solve them on your own and then check the solutions afterwards. Do not give up too quickly if you struggle to solve any of them. Move on to a different problem and then come back to the difficult one after a few hours.

- For the basic diff amp circuit depicted below, use a derivation like the one outlined in the supplemental reading “Real-World Performance of Difference Amplifiers” to show that a good approximation of the worst-case common-mode gain is given by the expression for A_{cm} given below. You may assume that the resistor value ratios satisfy the given approximation.



- The op-amp in the diff amp circuit depicted below has an output current magnitude limit of 10 mA. The resistor values used in the circuit are ridiculously small; this problem helps to illustrate why. Input voltage v_{in} can be as high as ± 250 mV before the output saturates at one of the power supply limits (± 10 V here). However, output current limiting prevents the output voltage from reaching that level. If $v_{in} = -250$ mV (DC), find the resulting voltage v across the input terminals of the op-amp (positive side of v at the “+” terminal). *Hint:* Check whether the load connected to the output of the circuit has a high enough equivalent resistance so that the current flowing through it can be considered negligible.

