Lesson 8: Summary

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1 Learning goals for this lesson

The following is a list of basic ideas that were covered during this lesson. You must be fairly conversant with all of them now. If you have any questions, please post them on Piazza. The link is `piazza.com/phys211`.

- Understand why objects rotate, in other words what produces rotation?
- Recognize the significance of the angle at which the force acts as well as how far from the "axis".
- Understand the reasons for the definition of torque, and the Newtons second law for rotation.
- Understand the origin of moment of inertia, and why it is dependent of the "mass distribution", or how far from the axis the mass is.

2 A quick summary

This section does not aim at being comprehensive. It's more of a slightly expanded version of the above, and primary serves to jog your memory, and be a quick look back place in case you forget something. If any of this is unclear, please post a question!

In the previous class, we learnt about how we can describe rotational motion, i.e, the rotation of a rigid object about a fixed axis. We obtained the rotational analogues of motion along a line, and came up with the "rotational kinematics" equations just like for motion in 1D. We carry on in an analogous fashion here. We ask, what produces rotational motion, and how do we quantify it.

Back when we studied motion in 1D, we recognized that motion was induced via an acceleration that produced a change in velocity, which in turn produced a change in position. In the case of rotation, we are looking for what might produce an angular acceleration. Of course an angular acceleration is just a convenient way of describing 2D motion in a circular path, and angular acceleration is essential a tangential acceleration, which is produced by a force acting on the object. What is important is that a force acting along the radius (line joining object and center of rotation) cannot produce any tangential acceleration, so it is only the tangential component of force that produces an angular acceleration. This is shown in fig. 1.

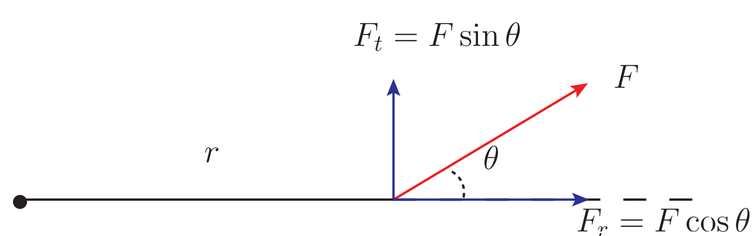

Figure 1: Only the tangential component of the force contributes to angular acceleration.

Now, we also understand from our daily experience of opening doors, that in order to produce any rotation, we need to apply a force away from the axis. In fact, the farther out we apply the force, the easier it is to produce rotation. In other words, the turning effect of our force is greater the farther we are from the axis. What we recognize as ease of producing rotation, is just how much angular acceleration we can produce. So, this leads us to think that the angular acceleration is proportional to the distance at which the force acts, as well as it's tangential component. We write,

$$\alpha \propto rF_t. \tag{1}$$

The above product on the right hand side is called torque (τ) or moment of force, and counter-clockwise torque conventionally has a positive sign. We need to now figure out what the proportionality constant is. For 1D motion, we called it the mass of an object. We expect it to be related, and we can find it out by treating rotational motion as 1D motion along a tangent (fig. 2. We get,

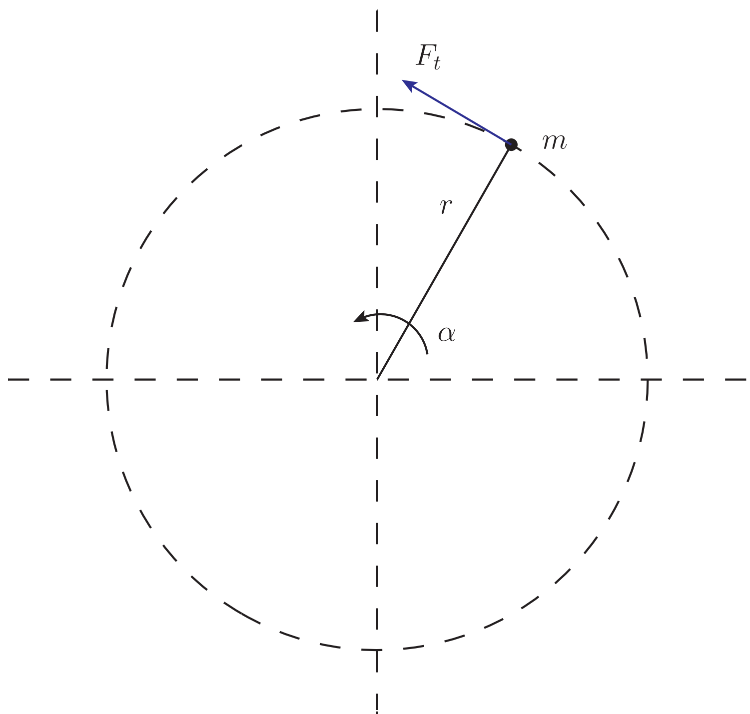

Figure 2: Trying to find the relation between angular acceleration and torque.

$$ma_t = F_t. \tag{2}$$

Multiplying by the radius r at which the force acts, we get,

$$ma_t r = rF_t = \tau \tag{3}$$

Writing $a_t = r\alpha$, we get,

$$mr^2\alpha = \tau. \tag{4}$$

So the angular acceleration is directly proportional to the torque, and inversely proprtional to a new quantity that depends on the mass of the object as well as the radius at which it sits! This tells us that the farther away the mass is from the axis, the harder it is to rotate, something we are all used to! The new combination

mr^2 is called the moment of inertia and denoted by I. We therefore end up with the rotational analogue of Newtons second law:

$$\tau = I\alpha. \tag{5}$$

The above example was a simple case of a single point object of mass m sitting at a radius r. Now that we're talking about rotation, we need to understand how extended objects behave. We'll restrict ourselves to the domain of what are called "rigid" objects. It means that the objects don't deform under any forces. Also the net force on the object is the sum of all forces acting on different parts. In some sense it behave like several tiny masses stuck together with super superglue. Naturally the angular acceleration is going to be given by the sum of all torques acting on the object (with the appropriate signs). For such an object the moment of inertia is given by the sum of the moments of inertia of the "tiny masses" that make it up. In the case of a continuous object, the sum becomes an integral (an integral is just a sum over many many very tiny pieces). It therefore depends on how the mass is distributed in an object. Below we'll find the moment of inertia of a thin rod with a uniform mass distribution (mass per unit length) about an axis going through it's end. The position of the axis is crucial since changing the axis of rotation changes the mass distribution around the new axis!

In figure. 3 is shown a thin rod. We want to compute the moment of inertia about an axis going through

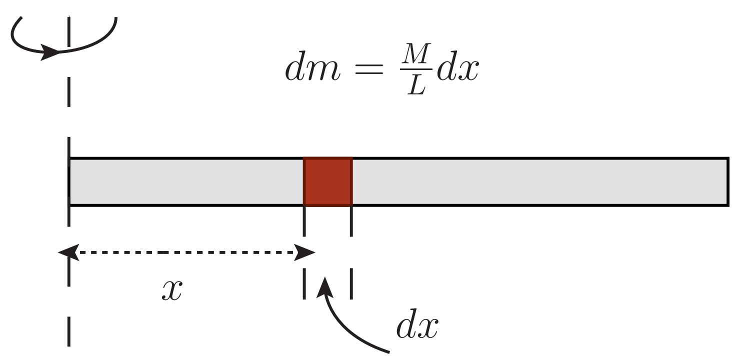

Figure 3: Thin rod with axis through the end

the end. We divide the road in several tiny masses dm of length dx. A tiny mass dm at distance x from the axis of rotation has a moment of inertia

$$dI = dm\, x^2. \tag{6}$$

For a uniform mass per unit length M/L, we have $dm = \frac{M}{L}dx$ giving,

$$dI = \frac{M}{L}dx\, x^2. \tag{7}$$

To find the total moment of inertia, we have to sum this over all the tiny pieces from $x = 0$ to $x = L$, in other words, do an integral.

$$I = \frac{M}{L}\int_0^L dx\, x^2 = \frac{ML^2}{3}. \tag{8}$$

So, the moment of inertia of the rod in the above example is a third of what it would have been if all the mass was concentrated at a distance of L. We couldn't have guessed the "third" without doing the above calculation, but it's certainly obvious that the moment of inertia should decrease (i.e. it should be easier to rotate the rod) since some of the mass is near the axis, some a bit far, and some farther out.

One can do the same exercise with a different axis of rotation. Play around with it and find out where you'll put an axis through it for the maximum ease of rotation!

3 Additional resources

- Look at the files area on angel for links.