Lesson 5: Summary

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1 Learning goals for this lesson

The following is a list of basic ideas that were covered during this lesson. You must be fairly conversant with all of them now. If you have any questions, please post them on Piazza. The link is `piazza.com/phys211`.

- What is weight of an object and gravitational acceleration;
- How do we understand systems with pulleys and strings;
- Kinematics in 2D - position, velocty and acceleration as vectors;
- Motion in 2D - example of projectile motion;
- Forces in 2D - addition of forces, resolving forces;
- Friction - measuring the coefficient of friction.

2 A quick summary

This section does not aim at being comprehensive. It's more of a slightly expanded version of the above, and primary serves to jog your memory, and be a quick look back place in case you forget something. If any of this is unclear, please post a question!

We started our discussion today where we left off. We got to this business of weight, and we'd like to understand it better. In general, all of us feel our weight due to gravity. Things fall due to gravity - they are attracted by the earth. It is interesting to note however, that gravity imparts the same acceleration to all objects (near the surface of the earth). A light object and a heavy object dropped from the same height take equal amounts of time to hit the ground. We saw some videos where this was tried with a feather and a stone in a vacuum - indeed they both hit the ground at the same time. Gravitational force, then, can be calculated using this experimental result as $F_g = mg$, where g is conventionally the letter used for acceleration due to gravity. It points downward (towards the earth's center) and has a value of about 9.8m/s^2 near the earth's surface. That means that a falling object increases its speed by about 10m/s every second. 10m/s is about 22mph, so in just a few seconds, it accelerates an object to a pretty high speed. So, as of now, we have this very rudimentary phenomenological view of gravity. It will suffice for now. What's important, again, is that the gravitational acceleration is the same for all objects.

With this, we can now talk about objects under the influence of gravity. A very simple system is a pulley, with two masses hanging at the ends of a taut massless, frictionless string. The pulley is also assumed to be massless and frictionless. These are simplifying assumptions and as we proceed gain a more sophisticated understanding of dynamics, we will lift some of these. A massless string implies that it has the same tension throughout it, even when it is accelerating. If it was massive, the tension would be different along its length to account for the acceleration of the mass of string. Friction in general complicates matters and will require invoking rotation of the pulley. We keep it out for now. Note that a string is a length constraint. It keeps the distance between the two objects it connects fixed. If the length is fixed, then the two objects must have the same velocity and acceleration (assuming the string doesn't stretch). The pulley then only serves to change the direction. Also important is that a string can only pull, not push. So all forces applied by a string (usually denoted by a tension, T), must be directed away from the object it acts on.

We'll solve for the acceleration of the two blocks shown in fig. 1 with the above assumptions. Fig. 1 also

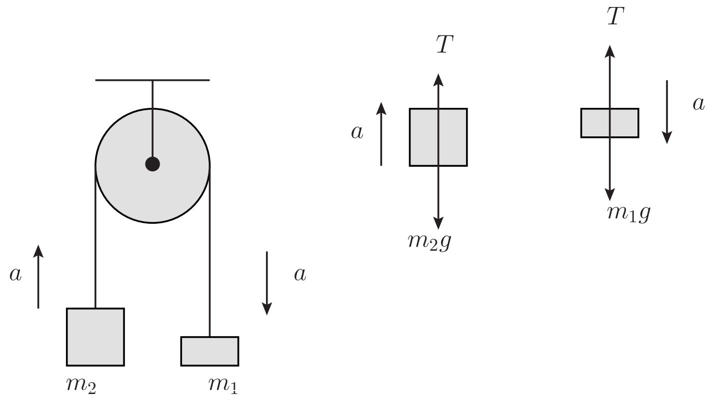

Figure 1: A simple massless frictionless pulley with masses hung on a frictionless massless string.

shows the free body diagrams for each of the blocks. Note that we've assumed already that the tensions are equal on both ends. Since the pulley is fixed, we don't worry about it's free body diagram. The fixed length of the string also allows us to assume that both blocks have the same magnitude of acceleration. We are free to assume a direction. The equations will tell us which way it goes. Writing down Newton's second law for each block, we get,

$$F_{\text{total},1} = m_1 g - T = m_1 a, \qquad F_{\text{total},2} = T - m_2 g = m_2 a. \tag{1}$$

Note that we've written the total forces in the direction of the acceleration already. Adding the above two equations, we get

$$a = g\frac{m_1 - m_2}{m_1 + m_2}. \tag{2}$$

We can also find T. Note the several limits of this problem that we intuitively understand. First, if the masses are equal, we don't expect them to move, and indeed for $m_1 = m_2$, $a = 0$. If one of the masses is 0, we expect the other to just fall freely, and this also can be shown by either setting m_1 or m_2 to 0. This also underscores the importance of using variables to solve the problem. If we had started with numbers, say 1kg and 3kg, we would get some answer, but we wouldn't know if it was right or wrong. Be using variables, we've solved the simple pulley problem for all possible masses!

We worked out a more complicated example of this, but seem to have run into some difficulties. We'll reexamine it in the next class.

We then went back to the beginning - understanding motion, but this time, in 2 dimensions. We saw that instead of giving one position as a function of time, we need to provide two positions with respect to two reference lines. We decided that it would be convenient to locate two positions with respect to two perpendicular lines, that then became our coordinate axes. The position then is given as $(x(t), y(t))$ with respect to some reference point which we denote (0,0) and call the origin of the "coordinate system". The point can equivalently be located by an arrow from the origin to the point, and we can specify the length of the arrow and the angle it makes with the horizontal, (r, θ). This is called a vector. The x and the y lengths are called the "components" of the vector. When we add vectors, we can add their corresponding components separately. Motion in 2 dimensions is then just like two separate motions each in one dimension put together. We denote position by $\vec{r}$ which indicates that it has magnitude (we write this as $|\vec{r}|$ or simply r) and direction. θ is conventionally the angle it makes with the x-axis. The components then are

$$x = r\cos\theta, \qquad y = r\sin\theta. \tag{3}$$

Velocity now, which is the derivative w.r.t time of position also gets two components.

$$v_x = \frac{dx}{dt}, \qquad v_y = \frac{dy}{dt}. \tag{4}$$

This can also be written as $\vec{v} = \frac{d\vec{r}}{dt}$. The acceleration similarly, gets two components,

$$a_x = \frac{dv_x}{dt}, \qquad a_y = \frac{dv_y}{dt} \qquad \text{or} \qquad \vec{a} = \frac{d\vec{v}}{dt}. \tag{5}$$

In 3 dimensions, we get a new z-coordinate and everything looks just like above, except that they each have **three** components. Note that position, velocity, and acceleration are really geometric quanities, with time thrown into the mix. Position is indeed a vector (satisfies rules of vector addition, e.g.), and the vector qualities are inherited by velocity and acceleration.

Let us try to understand the motion of an object in 2 dimensions under the influence of gravity. Firgure 2 shows an object being shot off at an angle θ with the horizontal and an initial speed v_0. Speed is another word for the magnitude of velocity. The components of the initial velocity, therefore are,

$$v_{0x} = v_0 \cos\theta, \qquad v_{0y} = v_0 \sin\theta. \tag{6}$$

Since the only force is downwards, producing a uniform acceleration of $-g$, the particle stays in a plane.

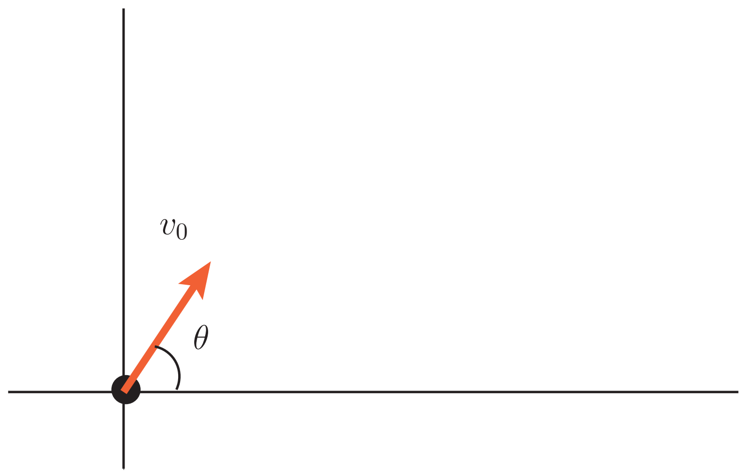

Figure 2: Projectile motion. Only gravity acts producing a vertically downward acceleration of 9.8m/s^2.

There are no lateral forces moving it out of a plane. We neglect air resistance also. With that, there are no forces in the x-direction. Therefore we have,

$$a_x = 0, \qquad a_y = -g. \tag{7}$$

There is only a constant downward acceleration. We can now treat the horizontal and vertical motion separately. In the horizontal direction,

$$v_x(t) = v_{0x} + a_x t = v_{0x} = v_0 \cos\theta, \tag{8}$$

and

$$x(t) = x_0 + v_{0x} t + \frac{1}{2} a_x t^2 = v_{0x} t = v_0 t \cos\theta. \tag{9}$$

The particle therefore has a constant x-velocity given by its initial value, and the position increases linearly with time. In the y-direction,

$$v_y(t) = v_{0y} + a_y t = v_{0y} - gt = v_0 \sin\theta - gt, \tag{10}$$

and

$$y(t) = x_0 + v_{0y}t + \frac{1}{2}a_y t^2 = v_{0y}t - \frac{1}{2}gt^2 = v_0 t \sin\theta - \frac{1}{2}gt^2. \tag{11}$$

The y-velocity on the other hand decreases linearly with time, until it hits zero, and then proceeds to become negative until it hits the ground again. The position increases and then falls back like a parabola. The plots in fig. 3 show x vs. t, y vs. t and y vs. x. The last plot is really the trajectory of the particle as you would see it. It is also a parabola. Given the initial speed and angle, therefore, we can predict where the object will fall, e.g., or how high it will get.

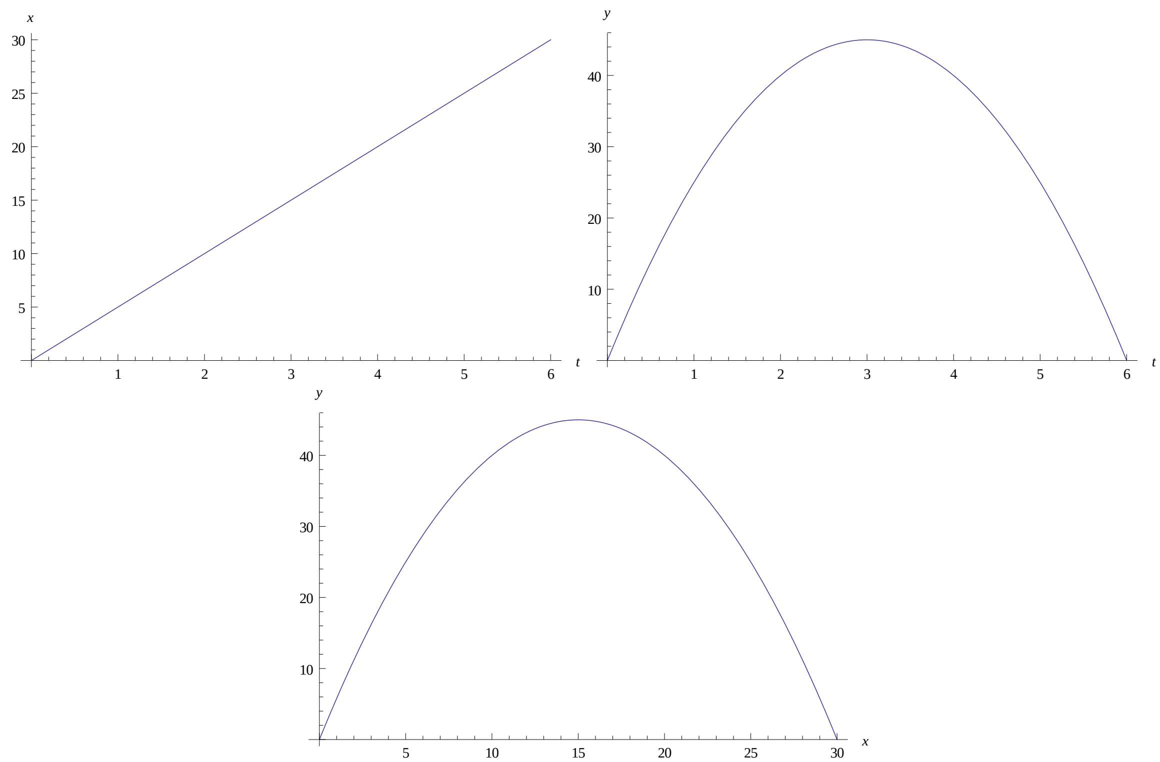

Figure 3: Plots of x vs. t, y vs. t and y vs. x for projectile motion. Look at the numbers on the axes and try to determine the initial velocity from these plots!

Now that we understand how to study the motion of an object in 2 or more dimensions, let's extend this discussion to the forces. How to forces behave in 2 dimensions? Now, we've already said that in 1 dimension, the force is given by the acceleration it produces on an object. We could directly generalize this to two dimensions, and say that the force produces acceleration along itself. In 2 dimensions, we can resolve this force into a force in the x-direction, F_x and a force in the y-direction, F_y and write,

$$F_x = ma_x, \qquad F_y = ma_y, \qquad or \qquad \vec{F} = m\vec{a}. \tag{12}$$

Writing it this way would imply that if multiple forces act from different directions on an object, the result is the **vector sum** of all forces. However, forces are real entities, and not mere descriptions, and so we have to ensure that forces indeed add in this fashion. This can be done by a cute little experiment where we either apply force in the x or the y direction, and measure their effects. We then apply both forces together and measure the new acceleration. If the above guess is correct, then we should be able to compare the vector sum from the first part with the measured result of the second part. They should be equal, and this is actually an important concept, that indeed we can add forces vectorially.

We can now put all of this together and include gravity in our box on a table problems. There is one important point here. Fig. 4 shows a box sitting on a table. Gravity acts on the box. It's weight mg, acts

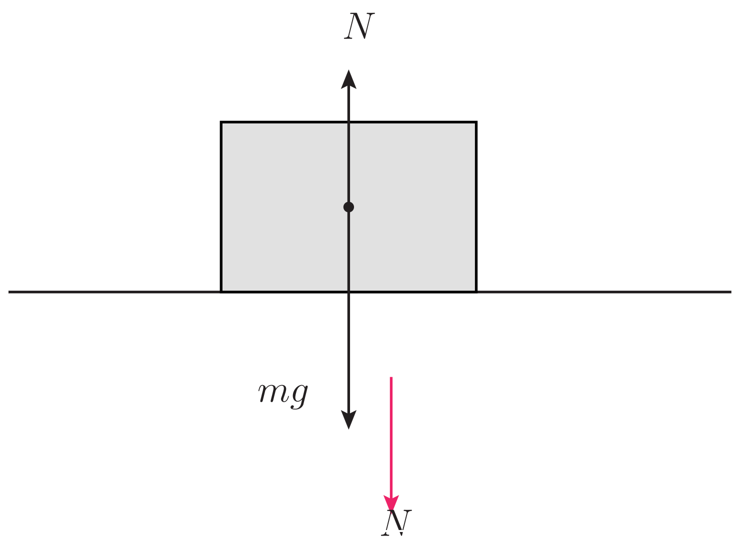

Figure 4: Box on a table. Gravity, and normal forces.

downwards on it, not on the table. However, this causes it to push against the table, thus producing some contact force N. The table responds to this contact force, and applies a force back on the box, also equal to N, by the third law. The net force acting on the box, therefore, is $F = N - mg$ in the upward direction. However, the box doesn't accelerate in that direction. So, $F = ma = 0$, implying that $N = mg$. Therefore, our intuition about us exerting our weight on a surface is correct. But really, it's a contact force we're exerting that happens to be equal to our weight. If we're on an incline, this changes.

The last thing we got to was friction, again. This time we qualified it a little better. In the last class, we said frictional force was proprtional to weight, but as we see above, it's not the weight that's exerted between the surfaces, but the contact force, and this depends on the angle of the surface w.r.t gravity! The frictional force therefore is proportional to the contact force, also called the "normal reaction". We therefore have,

$$F_{\text{max. friction}} \equiv f_{\text{max}} = \mu N. \tag{13}$$

Finally, we solve the problem of block sliding on an inclined plane with friction coefficient μ. The forces and the free body diagrams are shown in fig. 5 Note that the weight of the box acts vertically down, but the normal reaction acts perpendicular to the plane (hence the word "normal"). the acceleration of the block is along the plane and downwards. It is there natural to resolve the gravitational force. We resolve it into two directions - along, and perpendicular to the plane. In the vertical direction (i.e. perpendicular to the plane), the block doesn't move, so we have,

$$F_y = N - mg\cos\theta = 0 \implies N = mg\cos\theta. \tag{14}$$

In the horizontal (along the plane) direction,

$$F_x = ma = mg\sin\theta - \mu N = mg\sin\theta - \mu mg\cos\theta \implies a = g(\sin\theta - \mu\cos\theta). \tag{15}$$

With no friction, $\mu = 0$, and the block accelerates with $a = g\sin\theta$. Consider the following other situation: $\mu = 0.5$ and $\theta = 30°$. The acceleration is then approximately -10m/s^2. That means the block moves backwards, which doesn't make any sense. So it must be that the downward force of gravity is not yet large

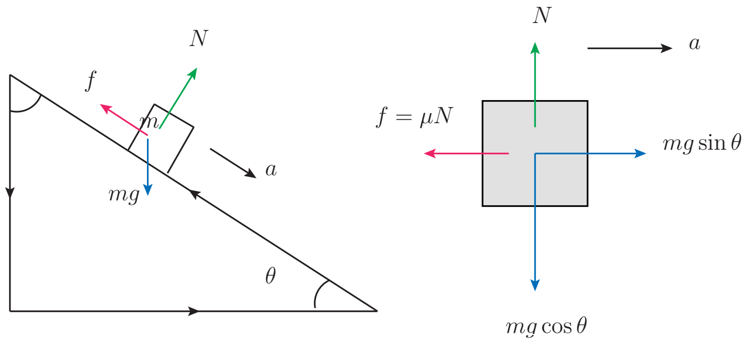

Figure 5: Block on an inclined plane.

enough to overcome friction, and friction is not at its maximum value. So the acceleration is 0 and the friction force $f = mg\sin\theta$. We can use this as an experiment to find out the value of μ. As soon as the block slips, we know friction has hit it's maximum value. But if we're right at the boundary of the block sliding, then we know that the downward force is approximately equal to $f_{\max}$ thus giving us,

$$\mu = \tan\theta, \tag{16}$$

θ now being this "critical" angle above which the block slides.

3 Additional resources

- Look at the files area on angel for links. Play with the PhET projectile sim.