Lesson 4: Force and motion in 1D

Deepak Iyer

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1 Learning goals for this lesson

The following is a list of basic ideas that were covered during this lesson. You must be fairly conversant with all of them now. If you have any questions, please post them on Piazza. The link is `piazza.com/phys211`.

- The notion of inertia;
- Force as the agent for motion - in particular, acceleration;
- $F = ma$ as relating force and the acceleration produce via the "inertia" m.
- Contact forces and reaction, the third law of motion.
- The origin of friction, and how we can model it.

2 A quick summary

This section does not aim at being comprehensive. It's more of a slightly expanded version of the above, and primary serves to jog your memory, and be a quick look back place in case you forget something. If any of this is unclear, please post a question!

In the previous lessons, we learnt how to describe motion of an object in 1 dimension. We specify the position x as a function of time t with respect to some reference point (also called the 0 or the origin). We are now interested in figuring out what produces motion in the first place, and if there's a way to quantify it.

Day to day experience tells us that it's harder to move some objects and easier to move others. In a lot of cases we're fighting friction, but even in relatively frictionless cases (like rolling a heavy barbell on the floor, or pushing a heavy shopping cart), we realize that the more there is the harder it is to move. We also intuitively understand the meaning of the word force - you apply a force to move something, and you apply a larger force to move something bigger.

This property of the object to be easy or difficult to move, we call "inertia". It of course has to to do with the mass of the object. For now, it's just a property. Another thing to realize is that we produce motion by accelerating objects. We do not directly impact its position or velocity, but only it's acceleration. Acceleration produces a change in velocity which in turn produces a change in position. We therefore would like to relate force and acceleration - this leads us to Newton's second law of motion:

$$F = ma. \tag{1}$$

m is called the "mass" of the object, and is a measure of how much force is required to give it a said acceleration. In other words, a unit force will give a unit mass a unit acceleration. In standard units, 1kg is given an acceleration of 1 m/s^2 by 1 Newton or 1 $\frac{\text{kg}\cdot\text{m}}{\text{s}^2}$. An acceleration of 1 m/s^2 in turn produces a change in speed of 1 m/s every second.

When several forces act on an object, the object accelerates under the influence of all of them together. In 1 dimension, forces all act along the same line. Forces in the same direction add, while forces in opposite directions subtract. The acceleration is always in the direction of the net force (i.e., total force). It is convenient to draw pictures when understanding the action of several forces. To keep things simple and straightforward, we draw forces by arrows in the direction of their action. We place the tail of the arrown inside the object (or at the point) on which the forces are acting, as shown in fig. 1

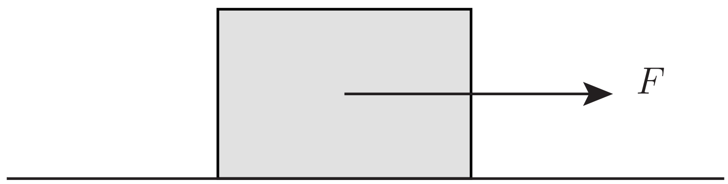

Figure 1: Representing forces pictorially.

We now need to understand how objects exert forces on each other. Most objects that we are familiar with interact via contact. An obvious exception is gravity, and we'll discuss that later. Most solids we are familiar with are impenetrable and resist being pushed against each other. We'll call these contact forces for now (their origin lies in electromagnetic forces). What's important is that if object A pushes on object B with force F, then object B pushes back on object A with force F. In order to keep the notation straigh F_{AB} would mean force by A on B. In a picture, this arrow would have it's tail sitting inside B. The "reaction" force, F_{BA} acts back on A and has it's tail sitting inside A. Thus the "action" and "reaction" forces act on different objects. This action-reaction principle is what is usually known as Newton's third law of motion.

When multiple objects push against each other, they transmit force via this contact force. Consider the following problem (fig. 2). A known external force F acts on the left block. Our task is to find the

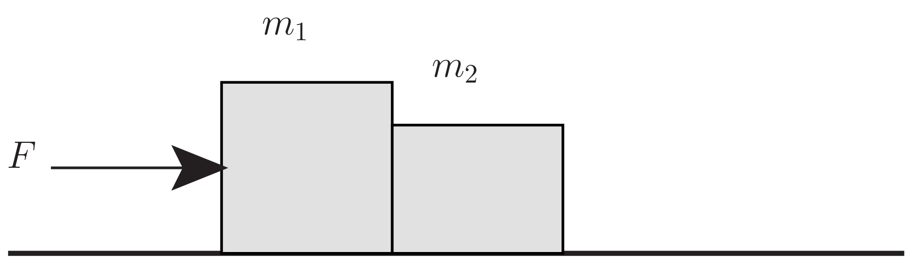

Figure 2: Two blocks being pushed.

acceleration of each block and the contact forces. If we try to think about what happens, it's fairly obvious that the two blocks move as a unit. One might even go further and say that to find the acceleration, we can replace the two blocks with one block of mass $m_1 + m_2$. While this might seem intuitively obvious, we can't provide a strong reason as to why it should behave like that. On the other hand, one tends to think of force being transmitted through solid objects without change. If that were the case, then each object would have a different acceleration, which is obviously not what happens. It is important to recognize the basis for our intuition. The only assumption that we can make about this system is that it "moves as a whole". That is not to say that it behaves like a big combined mass, but really to say that both blocks have the same acceleration. There is nothing else we can assume or state without reason about the forces between the blocks. We only have the two laws above to guide us. So let's begin. First, the external force F acts only on the left block. The left block pushes against the right block exerting some contact force, let's call it F_{12}, on it. The second block reacts to this and exerts force F_{21} back on block 1. By virtue of the third law, the magnitudes of these forces are equal. Shown in fig. 3 are the forces, as well as "free body diagrams" for each of the blocks. It is useful to separate the elements and show only the forces acting on them. That allows us to write down Newtons second law for each of the elements. We have for the first block,

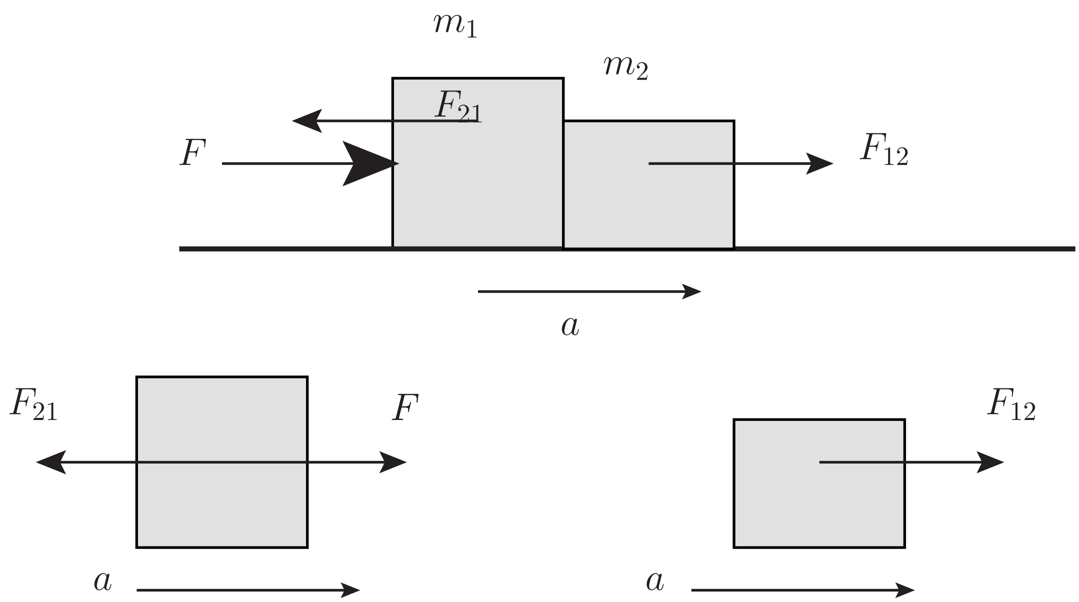

Figure 3: Forces and free-body diagrams.

$$F_{total,1} = F - F_{21} = m_1 a_1, \tag{2}$$

and for the second block,

$$F_{total,2} = F_{12} = m_2 a_2. \tag{3}$$

We assume that the blocks move together, allowing us to state that $a_1 = a_2$. Further, the third law allows us to state that $F_{12} = F_{21}$. Note that we've already taken into account the directions of these forces in the above equations. We therefore have,

$$F - m_2 a = m_1 a \implies a = \frac{F}{m_1 + m_2}. \tag{4}$$

We therefore obtain our intuitive result that the blocks move as a bigger mass $m_1 + m_2$, but into it went the assumption that Newton's third law held true. So it seems like our intuition is automatically wired for this. The same process can be carried out with three blocks. It's longer and the calculations are more tedious, but intuitively we expect that we should end up with the result $a = \frac{F}{m_1+m_2+m_3}$ for each block. Do this as an exercise.

The last part of this lesson had to do with introducting friction. Microscopically, friction is due to tiny ridges on surfaces interlocking and other attractive forces that emerge at molecular scales. While a true understanding of friction requires an understanding of the microscopic details of the surfaces, we can model it by doing experiments. Most commonly used surfaces behave similarly when it comes to friction. Frictional force always opposes motion, so it's always opposite to the direction of motion (i.e., velocity direction). It is a self-adjusting force that keeps increasing till it reaches it's maximum value, and then it stays there (or decreases slightly if motion starts). Its behaviour can be understood by trying to pull objects of different weights across different horizontal surfaces at constant speed, that the amount of force required (which is equal to the frictional force, since we're moving at constant speed - see fig. 4) is proportional to the weight of the object and depends on the surface. For horizontal surfaces, we can write

$$\text{max. frictional force} = \mu \times \text{weight} \tag{5}$$

where μ is a number that has to do with the two surfaces in contact, and is called the co-efficient of friction. More on this in the next set of notes.

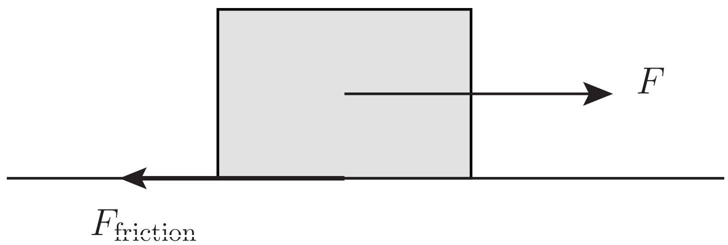

Figure 4: The external force and frictional force are equal for constant speed. For motion to start, the external force has to first become slightly greater than frictional force.

3 Additional resources

- Look at the files area on angel for links. See the *PhET* simulations for friction.