Lesson 2: Summary

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1 Learning goals for this lesson

The following is a list of basic ideas that were covered during this lesson. You must be fairly conversant with all of them now. If you have any questions, please post them on Piazza. The link is `piazza.com/phys211`.

- Using position and time to describe motion of an object (along a line)
- Understanding that knowing the position as a function of time (i.e. x and t for all time t) is sufficient information to completely describe the motion of an object.
- Representing this information on a plot of x vs. t.
- Why speed and acceleration are meaningful quantities to define
- Given an object moving at constant speed, how can we predict its position?
- Given an object moving with constant acceleration, how can we predict its position?

2 A quick summary

This section does not aim at being comprehensive. It's more of a slightly expanded version of the above, and primary serves to jog your memory, and be a quick look back place in case you forget something. If any of this is unclear, please post a question!

We started out this class by trying to describe the motion of an object. Several things came up during this discussion. First is that in order to properly describe the motion you need to be able to locate the object (or a specific point on the object - we'll not make a distrinction for now). In other words, we have to provide it's position, as a distance from some known reference point. The reference point is completely arbitrary as long as it stays fixed for the entire description. Motion then was described as a change in the position of the object with time. It therefore suffices to specify where and when the object is. For motion along a line, the position is denoted by the letter x, and time by t. Describing motion then involves giving a series of numbers like $(x_1, t_1), (x_2, t_2), \ldots$. Table1 shows an example, and fig 1 shows this represented as a plot. Plotted data is usually referred to as a *function*. The notation $x(t)$ is used, which means x at a given t.

We then recognize that we tend to intuitively describe motion in terms of speed and whether the object is speeding up or slowing down and so on. From our simple example of a car leaking oil at either fixed time intervals (every second, say) or fixed distance intervals (every meter, say), we determined that the separation of these points either in position or time were good measure of the speed, leading us to the slightly qualitative definition

$$\text{speed} = \frac{\text{separation in position}}{\text{time interval}}. \tag{1}$$

t	x
0	1
1	3
2	4
3	-1
4	2
5	4
6	5

Table 1: An example of position time data.

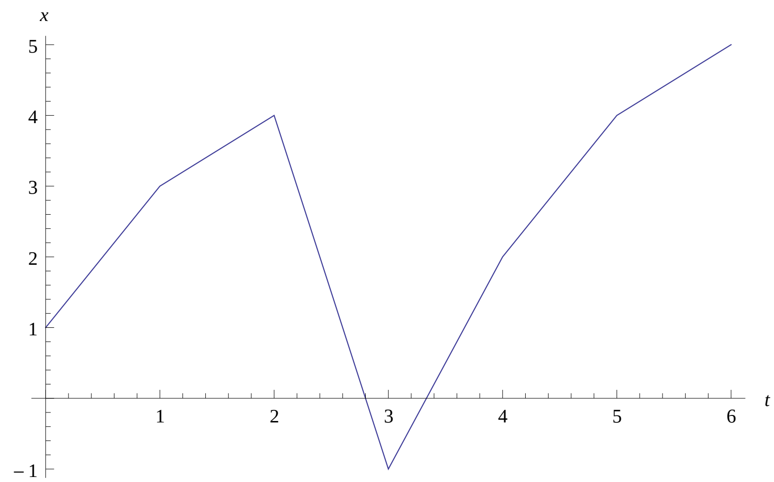

Figure 1: Plot of the above data

From here, we distilled a more sophisticated formula for speed. If the speed(v) is not changing, then it is given by

$$v = \frac{x_2 - x_1}{t_2 - t_1}, \tag{2}$$

where v determines the constant speed in interval $1 \to 2$. We can use the above formula to predict where an object will be if we know the speed (v), initial position (x_0) and time interval (t) by

$$x = x_0 + vt. \tag{3}$$

Note that the above is only true if the speed is constant. If it isn't, then we need to define it in a smaller interval of time where it is approximately constant. In general, this happens as the interval becomes really tiny, and we have

$$v \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}. \tag{4}$$

Thus the derivative enters naturally via the limiting process. Note that defined this way, the speed is just the slope of the x vs t plot at the point where we want to find the speed.

We tooks this a step further to align with our intuition of speeding up and slowing down and defined changes in velocity. Analogous to the position case, we define *acceleration* as the *rate of change of velocity* i.e., change in velocity per unit interval in time. We have,

$$a \equiv \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d}{dt}\frac{dx}{dt}. \tag{5}$$

The above definitions are in line with what our intuition tells us. We understand changes in position and changes in speed. It only makes sense to define these per unit interval in time. Working backwards, for an object undergoing constant accleration staring at position x_0 and velocity v_0 and time $t = 0$, we get,

$$dv = a\,dt \implies \int_{v_0}^{v} dv = \int_0^t a\,dt. \tag{6}$$

Since we take the acceleration a to be constant, we have

$$v(t) - v_0 = at \implies v(t) = v_0 + at. \tag{7}$$

Further,

$$dx = v\,dt \implies \int_{x_0}^{x} dx = \int_0^t v(t)\,dt = \int_0^t (v_0 + at)\,dt \tag{8}$$

Integrating, we get

$$x(t) = x_0 + v_0 t + \frac{1}{2}at^2. \tag{9}$$

The above equation lets us locate the particle given x_0,v_0 and a (for constant acceleration). These three together are called initial conditions.

While the latter bit involves a fair bit of math, one mustn't forget the simple origins of these definitions and to resolve any confusion, it's best to go back to the basics.

3 Additional resources

- Look at the files area on angel for links. I've posted the "moving man". You will need java to run this. It should be fairly easy to install.