Lesson 15: Summary

Deepak Iyer

June 12, 2012

1 Learning goals for this lesson

The following is a list of basic ideas that were covered during this lesson. You must be fairly conversant with all of them now. If you have any questions, please post them on Piazza. The link is `piazza.com/phys211`.

- Understand the origin and the meaning of the conservation laws.
- Recognize that the energy description is equivalent to the force description.
- Understand rotational energy, and angular momentum.

2 A quick summary

In the last lesson, we understood how kinetic energy and potential energy are naturally related to the work done by a force or by gravity in particular, respectively. We'll describe one more notion here, that of power. For some systems, which produce some form of energy (light bulbs, etc.) or do work continuously, the work they do is not a very good physical quantity that describes them since the amount of work done depends on the time for which this device is working. We therefore use the work done per unit time, and define that as **Power**. It has the units of J/s or Watts. A 100W light bulb therefore consumes 100 joules of energy every second.

We'll now talk a bit about the two conservation laws that we have come across. Conservation of linear momentum and conservation of energy. We saw in the lab that when the two carts collided, the total momentum before and after the collision was the same. How can we see this? For one, if we treat the two carts together as one system, then there was no external force on the system, and since, $F = \frac{\Delta p}{\Delta t} = 0$, we get $\Delta p =$. There is another way to see this, by treating each cart as a separate system. Let's call the initial momentum of cart 1 p_{1i} and it's final momentum (after collision) p_{1f} and similarly for cart 2. Then during the period of the collision, let's say the time for which contact occured was δt. If, during this period, cart 1 applied some (possibly complicated time dependent) force F_{12} on cart 2, and cart 2 applied F_{21} on cart 1, then we know from Newton's third law, that $F_{21} = -F_{12}$. Multiplying this equation by Δt, we get,

$$F_{21}\Delta t = -F_{12}\Delta t \tag{1}$$

But each side of this equation is the change in momentum of the object the force is acting on! So, we have,

$$\Delta p_1 = -\Delta p_2. \tag{2}$$

Writing this out in full, we get,

$$p_{1f} - p_{1i} = -(p_{2f} - p_{2i}), \tag{3}$$

and rearranging, we get,

$$p_{1i} + p_{2i} = p_{1f} + p_{2f} \tag{4}$$

thus giving us back the result above, that total momentum is conserved. We have assumed pretty much nothing other than the validity of Newton's second law of motion and his third law of motion. Momentum of a system is always conserved in the absence of external forces.

We've already come across the conservation of energy so I won't talk about it again. Together, the conservation of momentum and energy constitute fundamental laws in physics, as fundamental as Newton's second law, even though it might seem like we derived it *from* Newtons second law. While this is true, we can also obtain Newton's second law starting with the assumption of conservation of energy. We did this in class, and I leave this as an exercise. Do the block on wedge problem starting with the assumption that total energy is conserved, and proceed to differentiate the velocity along the inclined plane to find the acceleration of the block.

The concepts of momentum and kinetic energy can be extended to rotation objects as well. Analogous to linear momentum, we can define an angular momentum from a torque acting over some time period,

$$\int_i^f \tau \, dt = I(\omega_f - \omega_i) = I\omega_f - I\omega_i = L_f - L_i. \tag{5}$$

The angular momentum is denoted by L. We will treat it as a scalar quantity. Just like linear momentum is conserved in the absence of external forces, angular momentum is conserved in the absence of external torques! This has interesting consequences, since it is quite easy to change the moment of inertia of an object by changing the mass distribution. Then for L to remain fixed, ω will have to adjust. We'll see demos of this in class.

Another rotation quantity we can read off from it's linear counterpart is the rotational kinetic energy. For a given moment of inertia (about some axis), and rotation about that same axis with angular velocity ω, the kinetic energy is given by $\frac{1}{2}I\omega^2$.

3 Additional resources

- Look at the files area on angel for links.