Lesson 13: Summary

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1 Learning goals for this lesson

The following is a list of basic ideas that were covered during this lesson. You must be fairly conversant with all of them now. If you have any questions, please post them on Piazza. The link is `piazza.com/phys211`.

- Why is work a dot product between force and displacment?
- What is potential energy?

2 A quick summary

In the last lesson, we defined work and saw that the total work done by all the forces acting on an object was equal to the change in a quantity which we called the kinetic energy. However, our entire discussion was based in one dimension. How do we extend these quanities or define them properly when we're in two or three dimensions?

We defined work as the action of a force over some distance,

$$W = \int_i^f F\, dx. \tag{1}$$

If we're in two or three dimensions, we know that Force is a vector, and so is displacement. But, we need to be careful in understanding what we mean by the product of two vectors. One thing to note is that a force can only produce an acceleration in it's own direction. A force in the x direction, e.g., cannot influence the y motion of the particle. We're interested in calculating work done by a particular force (of choice), let's call it $\vec{F}$. If that force is perpendicular to the direction in which the particle moves (under the influence of all the forces acting on it), then it can't do any work! (See fig. 1). In general, for a force at some angle θ relative to the displacment, it is only the component of force that is parallel to the displacement that does work. The perpendicular component does not. If the angle between the force and the displacement is θ, then the relevant component of the force has a magnitude $F\cos\theta$. If the magnitude of the displacement is Δl, then the work done is

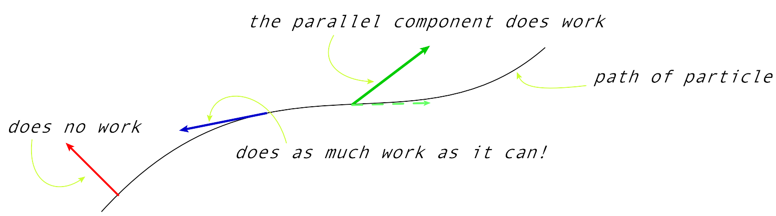

Figure 1: Component of force that does work is always parallel to the displacement.

$$W = F\cos\theta\,\Delta l. \tag{2}$$

The above is a way to define a product of two vectors $\vec{F}$ and $\Delta\vec{l}$, i.e., by choosing the component of one vector along the other, and multiplying their magnitudes. In vector notation, this is denoted by a "dot" and is called the "dot product" to differentiate it from another type of vector product, which we won't get into now. We write,

$$W = \vec{F} \cdot \Delta\vec{l}. \tag{3}$$

In general, if the force is varying over the path, then we write,

$$W = \int_i^f \vec{F} \cdot \Delta\vec{l}. \tag{4}$$

If the two vectors are denoted in vector notation as $\vec{F} = F_x\hat{\imath} + F_y\hat{\jmath}$ and $\Delta\vec{l} = \Delta x\hat{\imath} + \Delta y\hat{\jmath}$, then the dot product amounts to multiply the x-components and the y-components separately and adding the result. It can be shown by a simple picture to be equivalent to the above definition.

$$\vec{F} \cdot \Delta\vec{l} = F_x\Delta x + F_y\Delta y. \tag{5}$$

Thus, we see that the "correct" multiplication of vectors we need is dictated by the physics. Work is still a scalar quantity, and the above is a consistent way to multiply two vectors to get a scalar.

Now that we've taken care of that, we'll move on to something new. For this, we go back to our favorite problem, the block on the wedge. Consider a block held on a frictionless wedge at a height H above the table, as shown in fig. 2. We've solved this problem earlier and found the acceleration of the block. So, I'll write it directly. It is

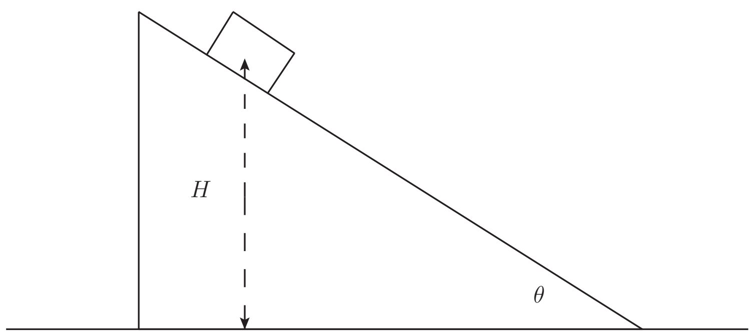

Figure 2: Revisiting the block on the wedge problem.

$$a = g\sin\theta. \tag{6}$$

We can now ask several questions. In view of what we're trying to get at, I'll ask, what is the velocity of the block when it gets to the bottom? We have quite some information. We know the acceleration, and we know the distance travelled. It is the length along the inclined plane L, given by,

$$L = \frac{H}{\sin\theta}. \tag{7}$$

We can now calculate the velocity at the bottom using,

$$v^2 = v_0^2 + 2a\Delta x = 0 + 2aL = 2g\sin\theta\frac{H}{\sin\theta} = 2gH \tag{8}$$

giving $v = \sqrt{2gH}$. The velocity at the bottom, therefore, doesn't depend on the mass, and it doesn't depend on the angle, only on the initial height above the table. This hints to us that there is something fundamental about an object under gravitational acceleration. We can calculate the work done by gravity. It is,

$$W_g = F_g\Delta x = mg\sin\theta\frac{H}{\sin\theta} = mgH. \tag{9}$$

This, is also independent of the angle, and only depends on the height. Since gravity was the only force acting, it must be reposinble for all the change in kinetic energy, so we have,

$$W_g = \Delta E_k = \frac{1}{2}mv^2 = mgH. \tag{10}$$

The initial kinetic energy was zero. Finally the height of the object above the table is zero. So it seems like as gravity does work, the kinetic energy increases, and the quantity mgh, where h is the height of the obect above the table at some time during the motion, decreases! Since being at some height gives the object the potential to move, we say that the object has **potential energy**, in this case, *gravitational* potential energy. The total energy is the sum of the potential energy E_p and the kinetic energy E_k, and it seems to be conserved! This is one of the fundamental principles of physics. Total energy is always conserved. If we have heat being generated (if there is friction), then some of the energy gets converted to heat, so the $E_p + E_k$ is not conserved, but the total energy in this case is $E_p + E_k + Q_f$ where Q_f is the heat given off (which is also equal to the work done by friction), and this is indeed conserved, as you have checked in class by adding friction to the above problem. In any problem where there isn't any friction, the sum of the potential and kinetic energies is always conserved.

3 Additional resources

- Look at the files area on angel for links. Check out the skateboard park PhET sim.