Lesson 12: Summary

Deepak Iyer

June 11, 2012

1 Learning goals for this lesson

The following is a list of basic ideas that were covered during this lesson. You must be fairly conversant with all of them now. If you have any questions, please post them on Piazza. The link is `piazza.com/phys211`.

- Understand how the impulse of a force is related to its change in momentum
- Why is momentum a useful quantity
- What is the effect of a force acting over a certain distance
- What is work, and how is it related to kinetic energy?

2 A quick summary

We now switch gears to talk about some different ways in which we can describe motion. Along the way, we'll stumble upon new terms, new physical quantities, new equations, and new laws of physics!

We begin by going back to lesson 2 or so where we thought about what produced motion, and how we could describe it. We hit upon Newtons second law of motion, i.e., $\vec{F} = m\vec{a}$. A force acting on a mass produces an acceleration that is proportional to the force and inversely proportional to the mass. However, in a lot of situations we are used to, like kicking a football or throwing a stone, the force that we apply is acting only for a brief period of time. Further, it is hard to be able to measure this force, since it varies in magnitude and direction over time. We can however, figure out its effect on the object. It accelerates it for a while, and then lets it go. If a force $\vec{F}$ acts over a short time Δt, then we can write down,

$$\vec{F} = m\vec{a} = m\frac{\Delta\vec{v}}{\Delta t} \tag{1}$$

Since we don't know the time interval exactly, we move it over to the other side, to get

$$\vec{F}\Delta t = m\Delta\vec{v}. \tag{2}$$

So, the effect of a force acting for a short period of time is to change the velocity of the particle. If the force acting changes over time, then we can divide it up, and say, that a force $\vec{F}_1$ acts for a time Δt_1 and so on. The total effect of this is,

$$\sum_i \vec{F}_i\Delta t_i = m\Delta\vec{v} \tag{3}$$

where $\Delta\vec{v}$ is now the total change in velocity produced by all these forces acting sequentially, i.e.

$$\sum_i \vec{F}_i\Delta t_i = m\vec{v}_f - m\vec{v}_i \tag{4}$$

As the time intervals become smaller, the sum on the left hand side becomes an integral. Also, we can write the right hand side as the difference in a new quanitity given by the product of mass and velocity, we'll call it $\vec{p}$. Since it is proportional to velocity, a vector, it is also a vector. We get

$$\int_{t_i}^{t_f} \vec{F}(t)\, dt = \vec{p}_f - \vec{p}_i \tag{5}$$

where $\vec{p} = m\vec{v}$. We'll give this quantity a name, **momentum** (not to be confused with **moment**). The total effect of a series of forces acting over some time period is then, to change the momentum of the particle. The advantage here, is that we skip all of the intermediate steps and directly reach the final momentum, and therefore final velocity. In a lot of cases, that's all we're interested in, so this is great!

The next thing we'll look at is the effect of forces acting over some distance. We'll do the remaining part of this lesson in one dimension for simplicity. The story for two or more dimensions will be part of the next lesson. While it might seem ad hoc to talk about forces over distances, there's a very good reason. Imagine pushing a heavy crate over a rough floor. The more force you have to apply (say the heavier, or more friction there is), the more you say you're working against something. The further you have to push the thing, again, the more "work" we do. This very intuitive notion of doing work having to do with applying forces and moving things is what we're talking about. We define the work done **by** a force F as

$$W = F\, \Delta x \tag{6}$$

where Δx is the displacement of the object under the influence of whatever forces are acting on it. Note that there might be several forces producing the displacement, but we can ask how much work is done by a particular force. For example, if five people push a crate, then we can ask how much work each of them did, while really, the crate was moving under the influence of all five forces. More about this in the next lesson. For now, we note an important thing, i.e., if there is no displacement, no work is done. If the force changes in time (or over the distance), then like before we need to add up all the little works from each segment, leading to,

$$W = \int_{x_i}^{x_f} F\, dx \tag{7}$$

Let's now assume that the object is moving only under the influence of F. Then, we can write down, for the motion of the object,

$$F = ma \tag{8}$$

Plugging this into into our work equation, we get

$$W = \int_{x_i}^{x_f} ma\, dx \tag{9}$$

This integral of course cannot be done the way it stands. We do not know anything about the acceleration. However, remember that acceleration was defined as $a = \frac{dv}{dt}$. Making this replacement, we get

$$W = \int_{x_i}^{x_f} m\frac{dv}{dt}\, dx \tag{10}$$

Now, we do a little trick. We write

$$\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dt} \tag{11}$$

where we've used $v = \frac{dx}{dt}$. Putting this into the above equation, we get,

$$W = \int_{x_i}^{x_f} mv\frac{dv}{dx}\, dx \tag{12}$$

The above integral can now be written as an integral over the velocity, from v_i to v_f which are the velocities corresponding to x_i and x_f. This gives us,

$$W = \int_{v_i}^{v_f} mv\, dv = \frac{1}{2}m(v_f^2 - v_i^2) = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2. \tag{13}$$

The work done then equals the difference in this new combination of mass and velocity, which we call **Kinetic Energy**. Kinetic, because it is energy due to motion (i.e. velocity). We thus have a nice result,

$$W = \Delta E_k \tag{14}$$

where $E_k = \frac{1}{2}mv^2$ is the kinetic energy. The result is called the **Work-Energy theorem** and states that the work done by a force equals the change in kinetic energy of the object, if this is the only force producing the motion.

We'll end here, and continue next time with work in higher dimensions and beyond.

3 Additional resources

- Look at the files area on angel for links.