CSCI 204: Data Structures & Algorithms

Revised by Xiannong Meng based on textbook author's notes

Binary Tree Application
Build Expression Tree
Heaps

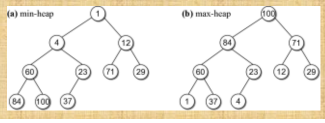
Revised based on textbook author's notes.

Heaps

- A heap is a complete binary tree in which the nodes are organized based on their data values.
- **heap order property** – how the nodes in a heap or arranged.
- **heap shape property** – as a complete binary tree.

Heap property, examples

- For each non-leaf node V,
  - **max-heap**: the value in V is greater than the value of its two children.
  - **min-heap**: the value in V is smaller than the value of its two children.

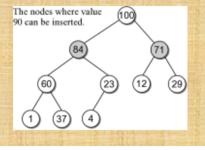


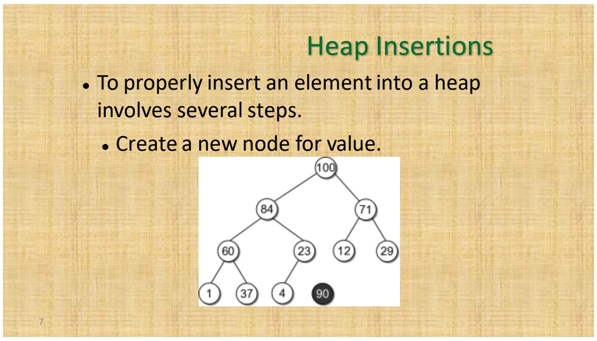
Heap Operations

- The heap is a specialized structure with limited operations.
  - Insert an element into the heap.
  - Remove the element from root node.

Heap Insertions

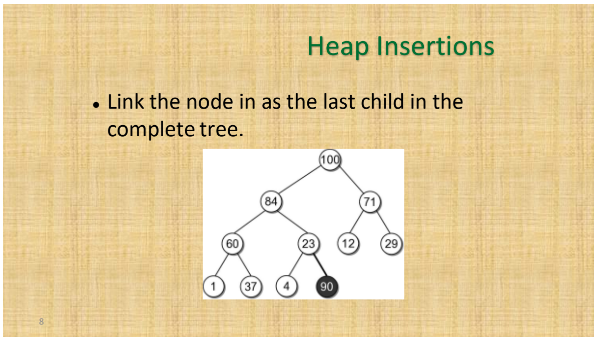
- When an element is inserted into a heap, both properties must be maintained.
  - Example: add 90 to the max-heap.
  - There are only 2 places where 90 can be inserted.





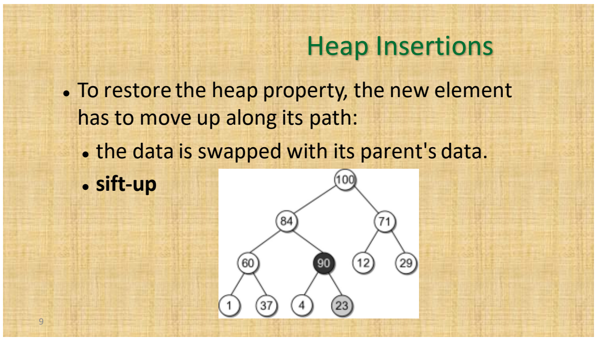
Heap Insertions

- To properly insert an element into a heap involves several steps.
  - Create a new node for value.



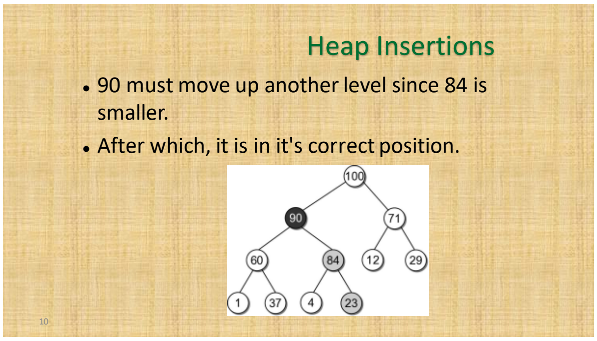
Heap Insertions

- Link the node in as the last child in the complete tree.



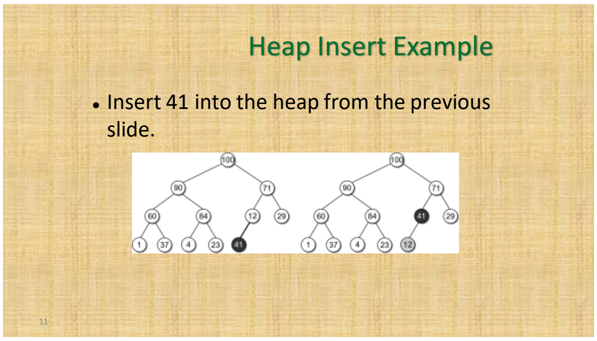
Heap Insertions

- To restore the heap property, the new element has to move up along its path:
  - the data is swapped with its parent's data.
  - **sift-up**



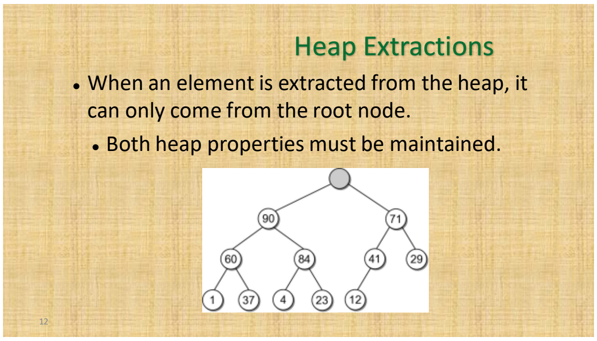
Heap Insertions

- 90 must move up another level since 84 is smaller.
- After which, it is in it's correct position.



Heap Insert Example

- Insert 41 into the heap from the previous slide.



Heap Extractions

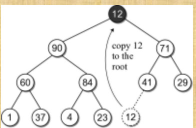
- When an element is extracted from the heap, it can only come from the root node.
  - Both heap properties must be maintained.

Heap Extractions

- To restore the tree to a heap:
  - another value will have to take the place of the extracted value in the root node.
  - a node has to be removed from the tree.

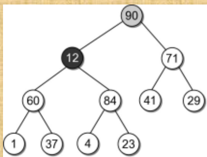
Heap Extractions

- There is only one node that can be removed and still maintain the heap shape property.
  - Copy the data from the last child node to the root.
  - Delete the last child node.



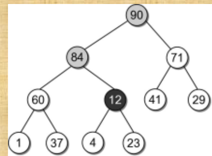
Heap Extractions

- To maintain the heap order property, the new root value has to be **sifted-down**.



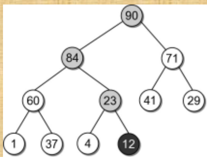
Heap Extractions

- The shifting continues until the value is placed into a leaf node, or it is larger than its children.



Heap Extractions

- After being swapped with 23, value 12 will be in a node that maintains the heap order property.


90
84
71
60
23
41
29
1
37
4
12