PHYS 212 Third Hour Exam
April 14, 2005

Name Key
Problem Session:
Hr _____ **Instr** _____

Show all work for full credit! Answers must have correct units and appropriate number of significant digits. For all the problems (except for multiple choice questions), start with some combination of (a) a fundamental equation; (b) a sentence explaining your approach; or (c) a sketch.

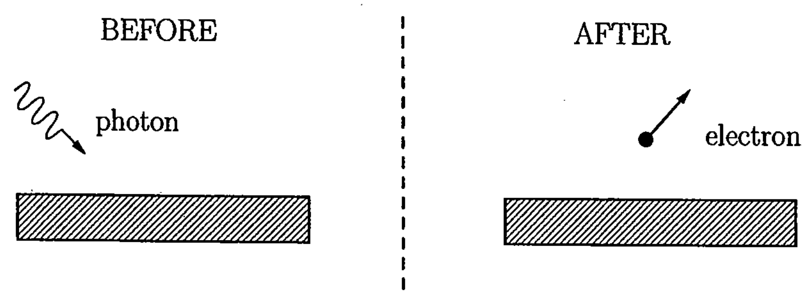
$c = 3.0\times10^8$ m/s $\quad k = 8.99\times10^9$ Nm^2/C^2 $\quad \mu_0 = 4\pi\times10^{-7}$ N/A^2

$h = 6.63\times10^{-34}$ Js $= 4.14\times10^{-15}$ eVs $\quad hc = 1240$ eVnm $\quad 1\text{eV} = 1.6\times10^{-19}$ J

$m_p = 1.67\times10^{-27}$ kg $= 938$ MeV/c^2 $\quad m_e = 9.11\times10^{-31}$ kg $= 0.511$ MeV/c^2

$m_n = 1.67\times10^{-27}$ kg $= 940$ MeV/c^2

1. (20 pts) Light with a frequency of 5.0×10^{14} Hz is incident on a metal surface with a work function of 1.0 eV, and electrons are ejected from the surface. A single photon interaction with the surface (cross-hatched) is illustrated in the figure.



a) Calculate the energy of a **photon** in the incident beam.

$E_{ph} = hf = 4.14\times10^{-15}\,\text{eVs} * 5\times10^{14}\,\text{s}^{-1} = 2.07\,\text{eV}$

or $6.63\times10^{-34}\,\text{J}\cdot\text{s} \cdot 5\times10^{14}\,\text{s}^{-1} = 3.32\times10^{-19}\,\text{J}$

b) Calculate the maximum kinetic energy for an ejected **electron**.

$K_{max} = E_{ph} - \Phi = 2.07\,\text{eV} - 1.00\,\text{eV} = 1.07\,\text{eV}$

or in J: $1.07\,\text{eV} \cdot \frac{1.6\times10^{-19}\,\text{J}}{\text{eV}} = 1.71\times10^{-19}\,\text{J}$

c) Calculate the maximum momentum for an ejected **electron**.

$K = p^2/2m \rightarrow p = \sqrt{2mK}$ Energy in SI units

$\Rightarrow p = \sqrt{2\cdot 9.1\times10^{-31}\,\text{kg} \cdot 1.71\times10^{-19}\,\text{J}} = 5.58\times10^{-25}\,\text{kg m/s}$

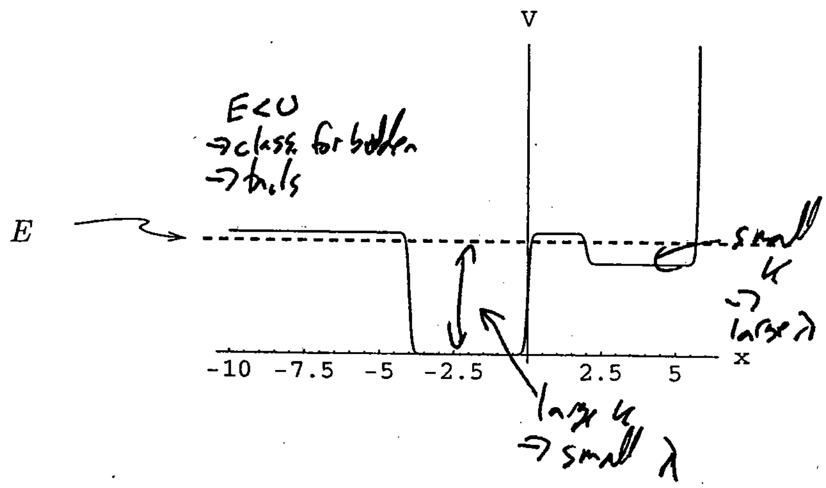
d) Calculate the minimum wavelength for an ejected **electron**.

$\lambda = h/p = \dfrac{6.63\times10^{-34}\,\text{Js}}{5.58\times10^{-25}\,\text{kg m/s}} = 1.19\times10^{-9}\,\text{m}$

or

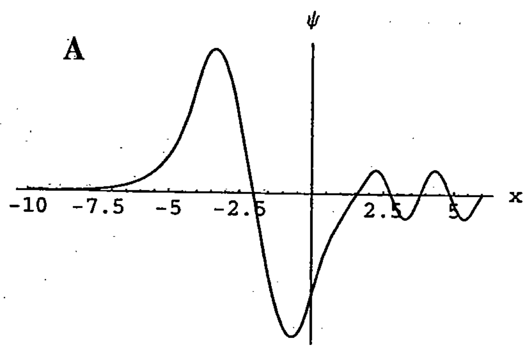
$1.19\,\text{nm}$

2. (10 pts) A one-dimensional potential energy curve ($V(x)$ vs. x) is plotted at right, and the total energy E of a particle is indicated by the horizontal dotted line. Which of the five graphs below represents a possible wavefunction for this particle? (Circle one.)

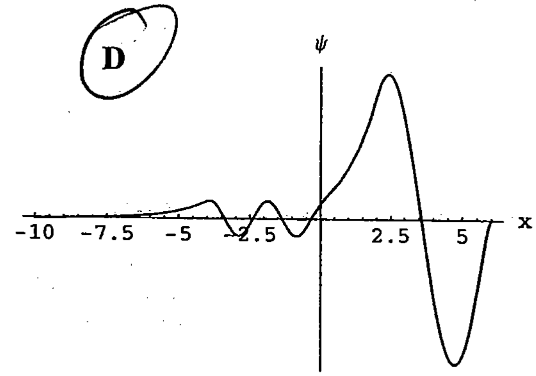


A B C D E

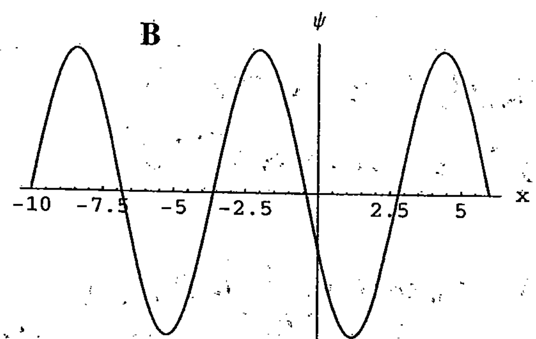
A



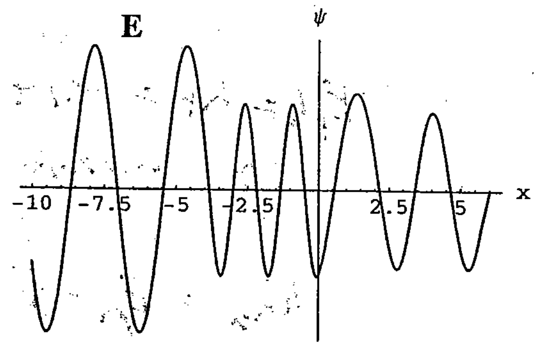
D



B



E



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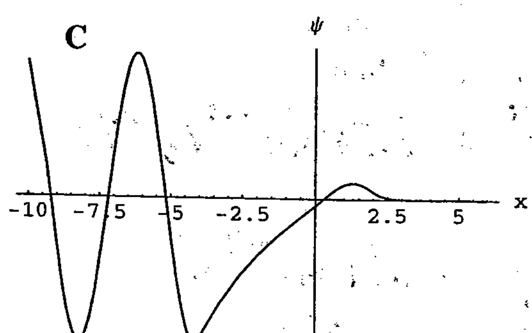


3. (16 pts) Consider the following spin states of an electron:

$$|+y\rangle = \sqrt{\frac{1}{2}}|+z\rangle + i\sqrt{\frac{1}{2}}|-z\rangle$$

$$|-y\rangle = \sqrt{\frac{1}{2}}|+z\rangle - i\sqrt{\frac{1}{2}}|-z\rangle$$

$$|\psi\rangle = \sqrt{\frac{1}{3}}|+z\rangle - i\sqrt{\frac{2}{3}}|-z\rangle$$

where the notation for spin states is that used in class.

a) A single electron is prepared in the state $|\psi\rangle$ given above. A subsequent measurement is made of the z-component of the intrinsic angular momentum (or spin) of this single electron. Calculate the probability this measurement will find the electron to have $s_z = +\hbar/2$. $\Rightarrow$ THAT IS, THE PROBABILITY OF MEASURING THE ELECTRON IN STATE $|+z\rangle$.

$$\text{PROB} = \left|\sqrt{\tfrac{1}{3}}\right|^2 = \underline{\underline{\tfrac{1}{3}}} \quad \left(= |\langle +z|\psi\rangle|^2 = \left|\left(\sqrt{\tfrac{1}{3}}\,\underbrace{\langle +z|+z\rangle}_{1} - i\sqrt{\tfrac{2}{3}}\,\underbrace{\langle +z|-z\rangle}_{0}\right)\right|^2 = \left|\left(\sqrt{\tfrac{1}{3}}\right)\right|^2 = \tfrac{1}{3} \right)$$

SINCE $|+z\rangle$, $|-z\rangle$ are ORTHONORM

b) A single electron is prepared in the state $|\psi\rangle$ given above. A subsequent measurement is made of the y-component of the intrinsic angular momentum (or spin) of this single electron. Calculate the probability this measurement will find the electron to have $s_y = -\hbar/2$.

$$\text{PROB}: |\langle -y|\psi\rangle|^2 = \left|\left(\sqrt{\tfrac{1}{2}}\langle +z| + i\sqrt{\tfrac{1}{2}}\langle -z|\right)\left(\sqrt{\tfrac{1}{3}}|+z\rangle - i\sqrt{\tfrac{2}{3}}|-z\rangle\right)\right|^2$$

$$= \left|\left(\sqrt{\tfrac{1}{6}}\,\underbrace{\langle +z|+z\rangle}_{1} - i\sqrt{\tfrac{2}{6}}\,\underbrace{\langle +z|-z\rangle}_{0} + i\sqrt{\tfrac{1}{6}}\,\underbrace{\langle -z|+z\rangle}_{0} - i^2\sqrt{\tfrac{2}{6}}\,\underbrace{\langle -z|-z\rangle}_{1}\right)\right|^2$$

$$= \left|\sqrt{\tfrac{1}{6}} + \sqrt{\tfrac{1}{3}}\right|^2 = \underline{\underline{0.97}}$$

4. (12 pts) Consider an electron that is confined in a "particle-in-a-box" potential with length $L = 1.0$ nm. Calculate the longest wavelength of light that can be emitted when the electron makes a transition ending in the first excited state.

$$E_n = \frac{n^2h^2}{8mL^2}$$

longest wavelength $\Leftrightarrow$ smallest energy jump

first excited state $\Leftrightarrow$ $n=2$

$\Rightarrow \Delta E = E_3 - E_2$ = energy of longest-wavelength photon emitted by electron ending up in $n=2$

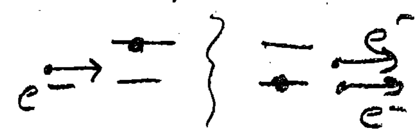
$$E_3 - E_2 = \frac{9h^2}{8mL^2} - \frac{4h^2}{8mL^2} = \frac{5h^2}{8mL^2} = \frac{5(6.63\times10^{-34})^2}{8(9.11\times10^{-31}\,\text{kg})(1\times10^{-9}\,\text{m})^2} = 3.02\times10^{-19}\,\text{J} = 1.88\,\text{eV}$$

$$(E_3 - E_2) = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{(E_3 - E_2)} = \frac{1240\,\text{eV}\cdot\text{nm}}{1.88\,\text{eV}} = \underline{\underline{659\,\text{nm}}}$$

5. (9 pts) An inventor approaches you with a proposal for a new electron-beam weapon. She proposes to build a "electron-aser" which would emit an powerful electron beam analogous to the photon beam of a laser. Some radioactive nuclei emit electrons when they jump from an excited energy level to a lower-energy level. This inventor suggests: make a sample of radioactive nuclei with a population inversion, and then electrons that are emitted from one radioactive decay will stimulate the emission of additional electrons. With the right electric fields at the ends we can reflect the electrons back into the sample to stimulate even more emission of electrons, thereby creating the "electron-aser." Should you invest money in this scheme? Explain why or why not.

Electrons are fermions. This means they must obey Pauli Exclusion Principle. Therefore stimulated emission of electrons (the creation of an e^- in a state identical to that of the incident e^-) is impossible. Don't invest!!

Proposal



6. (10 pts) A neon atom has 10 electrons. Assuming that the atom is in the state with the lowest possible total energy, write down the combinations of the quantum numbers n, l, m_l, and m_s for each of the 10 electrons.

	Electron	n	l	m_l	m_s
$1s^2$	1	1	0	0	$+1/2$
	2	1	0	0	$-1/2$
$2s^2$	3	2	0	0	$+1/2$
	4	2	0	0	$-1/2$
$2p^6$	5	2	1	1	$+1/2$
	6	2	1	1	$-1/2$
	7	2	1	0	$+1/2$
	8	2	1	0	$-1/2$
	9	2	1	-1	$+1/2$
	10	2	1	-1	$-1/2$

7. (9 pts) A single electron is confined to a one-dimensional infinite square well potential. It is in the superposition state $|\psi\rangle = \sqrt{\frac{2}{5}}|1\rangle + \sqrt{\frac{3}{5}}|2\rangle$, where $|1\rangle$ and $|2\rangle$ refer to the ground state and first excited state respectively, with energies $E_1 = h^2/(8mL^2)$ and $E_2 = h^2/(2mL^2)$. **Before** you make any measurements of this single electron, what can you say about its energy? (Circle one.)

a) The energy is E_1.

b) The energy is E_2

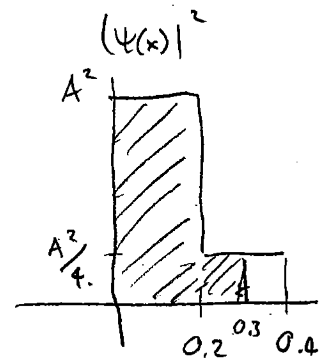
c) The energy is $E_2 - E_1$.

d) The energy is $\frac{(E_2 + E_1)}{2}$.

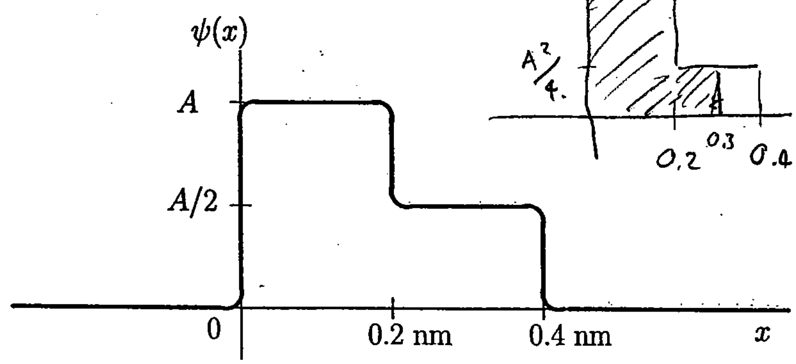
e) The energy is $\sqrt{\frac{2}{5}}E_1 + \sqrt{\frac{3}{5}}E_2$

f) The energy is $\frac{2}{5}E_1 + \frac{3}{5}E_2$

g) None of the other choices.



8. (14 pts) An electron is confined to move along the x-axis by some potential, resulting in the illustrated wave function $\psi(x)$:



a) Determine a numerical value for the constant A.

Need $\int |\psi(x)|^2 dx = 1$

So area under $|\psi(x)|^2$ vs $x = 1$

$\Rightarrow \quad 0.2\left(A^2 + \frac{A^2}{4}\right) = 1 \quad \Rightarrow \quad \frac{A^2}{4} = 1 \Rightarrow A = 2.$

b) Calculate the probability that a measurement of the position of the electron will find the electron in the interval $0.0\,\text{nm} \le x \le 0.3\,\text{nm}$.

Prob $= \int_0^{0.3} |\psi(x)|^2 dx$ = shaded area above

$= A^2\, 0.2 + \frac{A^2}{4}\, 0.1$

$= 0.8 + 0.1 = 0.9$