PHYS 212 Third Hour Exam
April 20, 2004

Name ______________
Problem Session:
Hr _____ Instr _____

<u>Show all work for full credit</u>! Answers must have correct units and appropriate number of significant digits. For all the problems (except for multiple choice questions), start with either (a) a fundamental equation (b) a sentence explaining your approach; or (c) a sketch.

$c = 3.0 \times 10^8$ m/s	$hc = 1240$ eV·nm	$h = 6.6\times10^{-34}$ J·s $= 4.14\times10^{-15}$ eV·s
$\mu_0 = 4\pi \times 10^{-7}$ N/A^2	$\mu_e = 9.28 \times 10^{-24}$ J/T	$\mu_p = 1.41 \times 10^{-26}$ J/T
$(E_1)_{\text{hydrogen}} = -13.6$ eV		1 eV $= 1.6 \times 10^{-19}$ J

1. (12 pts) A laser emits a continuous light beam with wavelength of 488 nm and a power of 1 mW. The entire beam is incident on a clean metal surface with a work function $\varphi = 2.1$ eV.

a) Calculate the maximum value of the kinetic energy of an individual electron ejected from the surface.

Use Photo-electric effect : $E_{ph} = \frac{hc}{\lambda} = \frac{1240\,eV\cdot nm}{488\,nm}$

$= 2.54\,eV$

$K_{max} = E_{ph} - \phi$

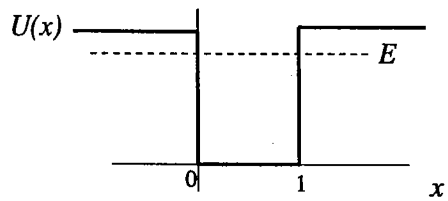
$= 2.54eV - 2.1$

b) Will an infrared laser that emits light with a wavelength of 1060 nm cause electrons to be ejected from the surface? Justify your answer quantitatively.

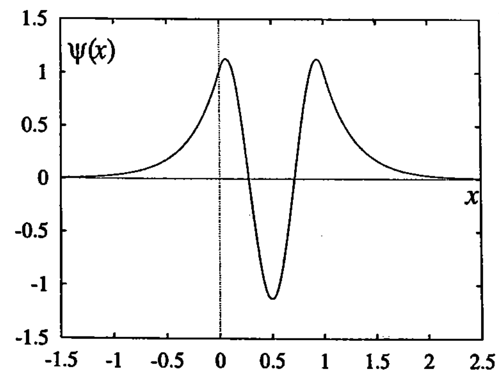
NO $\quad E_{ph} = \frac{hc}{\lambda} = \frac{1240}{1060} = 1.17eV$

This is less than ϕ, the minimum energy to release an electron.

2. (13 pts) Consider a particle with mass m in the illustrated one-dimensional *finite* square well. The normalized wave function for this particle is



$$\psi(x) = \begin{cases} 1.02\, e^{3.48x} & \text{for } x \le 0 \\ -1.13 \cos(7.20x - 3.60) & \text{for } 0 < x < 1 \\ 33.04\, e^{-3.48x} & \text{for } x \ge 1 \end{cases}$$



If the position of the particle is measured, calculate the probability that the particle will be found to the left of the origin, that is, in the region $x \le 0$.

$$P = \int_{-\infty}^{0} |\psi(x)|^2 dx = \int_{-\infty}^{0} (1.02)^2 e^{6.96x}$$

$$= 1.02^2 \left(\frac{1}{6.96} e^{6.96x} \Big|_{-\infty}^{0} \right)$$

$$= \frac{1.02^2}{6.96} = .1495$$

$$= \boxed{14.95\%}$$

3. (14 pts) Electrons in the (fictitious) atom Bucknellium have three (and only three) energy levels, as illustrated. Calculate the longest wavelength of light that can be emitted by Bucknellian atoms.

Long $\lambda \Rightarrow$ smallest ΔE

−1 eV

$\Delta E = 2\,eV = \frac{hc}{\lambda} = \frac{1240\,eV\cdot nm}{\lambda}$

−4 eV

$\boxed{\lambda = 620\,nm}$

−6 eV

4. (12 pts) Consider the solutions of the Schrödinger equation for the hydrogen atom.

a) How many hydrogen atom electron states have the principal quantum number $n = 2$?

$n = 2 \Rightarrow l = 0$ or 1

Possible nlm combinations:

$$\left.\begin{array}{lll} n=2 & l=0 & m=0 \\ n=2 & l=1 & m=+1 \\ n=2 & l=1 & m=0 \\ n=2 & l=1 & m=-1 \end{array}\right\}$$

2 spin states for each

$\Rightarrow$ 8 states

b) Find all possible values for the *magnitude* of the orbital angular momentum for electrons in states with the principal quantum number $n = 2$?

$$|L| = \sqrt{l(l+1)}\,\hbar$$

$$l = 0 \Rightarrow |L| = \sqrt{0(0+1)}\,\hbar = 0$$

$$l = 1 \Rightarrow |L| = \sqrt{1(1+1)}\,\hbar = \sqrt{2}\,\hbar$$

5. (14 pts) Consider a proton in a uniform magnetic field $\vec{B} = 1.5\hat{k}$ T. The proton is prepared to be in the state

$$|\psi\rangle = 0.6|+z\rangle + c|-z\rangle,$$

where c is a constant.

a) Determine a possible value for the constant c.

Normalization: $(0.6)^2 + c^2 = 1 \Rightarrow c^2 = 0.64$ $\boxed{c = 0.8 \text{ or } -0.8}$

b) 10,000 protons all prepared to be in the state $|\psi\rangle$. If measurements are made of the z-component of the spin angular momentum of these protons, approximately how many will be found to have a z-component of $+\hbar/2$?

Prob $(+\frac{\hbar}{2}) = 0.36$

For 10,000 protons: $(.36)(10{,}000) = \boxed{3{,}600}$

c) Determine the expectation value of a measurement of the z-component of spin angular momentum of a proton that has been prepared in state $|\psi\rangle$.

$$\langle S_z \rangle = (0.6)^2 \left(+\frac{\hbar}{2}\right) + (0.8)^2 \left(-\frac{\hbar}{2}\right) = \boxed{-0.14\hbar}$$

probabilities

values of S_z

6. (8 pts) Explain why a laser requires population inversion.

A laser is building up a population of photons in the same state via ~~coherent~~ stimulated emission, which requires at least some atoms to be at the higher energy level. However, atoms at the lower energy level absorb these photons, so we not only need some excited atoms, we need more than half to be excited $\Rightarrow$ population inversion

7. (14 pts) Consider a particle with energy $E = \frac{1}{2}U_0$ in a region where the potential energy is $U(x) = U_0$, where U_0 is a constant. Determine whether the function

$$\psi(x) = A\,e^{-\frac{\sqrt{mU_0}}{\hbar}x}$$

is a solution of the Schrödinger equation in this region. **You must show work.**

Schrödinger equation: $-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + U(x)\,\psi(x) = E\,\psi(x)$

$$\psi'(x) = A e^{-\frac{\sqrt{mU_0}}{\hbar}x}\left(-\frac{\sqrt{mU_0}}{\hbar}\right)$$

$$\psi''(x) = A e^{-\frac{\sqrt{mU_0}}{\hbar}x}\left(\frac{mU_0}{\hbar^2}\right)$$

$$-\frac{\hbar^2}{2m}\psi'' + U(x)\psi(x) \stackrel{?}{=} E\psi(x)$$

$$-\frac{\cancel{\hbar^2}}{2\cancel{m}}\cancel{A}\cancel{e^{-\frac{\sqrt{mU_0}}{\hbar}x}}\left(\frac{\cancel{m}U_0}{\cancel{\hbar^2}}\right) + U_0\cancel{A}\cancel{e^{-\frac{\sqrt{mU_0}}{\hbar}x}} \stackrel{?}{=} \frac{U_0}{2}\cancel{A}\cancel{e^{-\frac{\sqrt{mU_0}}{\hbar}x}}$$

After cancellation of common factors:

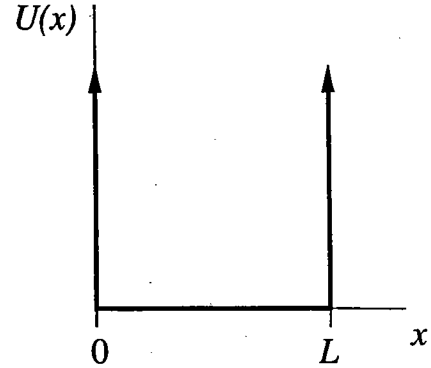
$$-\frac{1}{2}U_0 + U_0 \stackrel{?}{=} \frac{U_0}{2}$$

$$\frac{1}{2}U_0 = \frac{1}{2}U_0 \quad \checkmark$$

Based on the results of your work, do you conclude that $\psi(x)$ as given above is a solution? ~~Circle one:~~

IS A SOLUTION **IS NOT A SOLUTION**

8. (13 pts) Consider a particle of mass m in an *infinite* square well potential (particle-in-a-box) extending from $x = 0$ to $x = L$.



a) The graph for $\psi(x)$ drawn to the right is the wavefunction for (circle one):

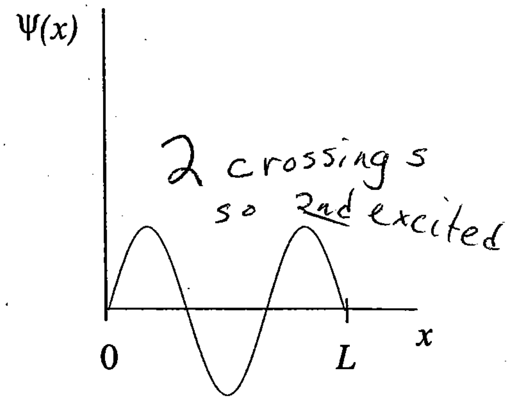
i) the ground state

ii) the first excited state

iii) the second excited state

iv) the third excited state

v) none of the above



b) Determine the wavelength for a particle in this state. Express your answer in terms of the given symbols and physical constants.

From diagram $3(\frac{1}{2}\lambda) = L$

So $\boxed{\lambda = \frac{2}{3}L}$

c) Determine the magnitude of the momentum of a particle in this state. Express your answer in terms of the given symbols and physical constants.

DeBroglie: $p = h/\lambda = \boxed{\frac{3h}{2L}}$