PHYS 212E | **Name ____________________**
Third Hour Exam | **April 22, 2004**

<u>Show all work for full credit</u>! Answers must have correct units and appropriate number of significant digits. For all the problems (except for multiple choice questions), start with either (a) a generally applicable equation or statement; (b) a sentence explaining your approach; or (c) a sketch.

<u>electron</u>	<u>proton</u>
$m = 9.11\times10^{-31}\text{kg} = 511\text{keV}/c^2$	$m = 1.67\times10^{-27}\text{ kg} = 938\text{MeV}/c^2$
$\mu_z = 9.27\times10^{-24}\text{J/T} = 5.8\times10^{-5}\text{eV/T}$	$\mu_z = 1.41\times10^{-26}\text{J/T} = 8.8\times10^{-8}\text{eV/T}$

$c = 3.0\times10^8\text{ m/s} = 3.0\times10^{17}\text{ nm/s}$ $\quad hc = 1240\text{ eV}\cdot\text{nm}$ $\quad 1\text{ eV} = 1.6\times10^{-19}\text{ J}$

$h = 6.626\times10^{-34}\text{ J}\cdot\text{s} = 4.136\times10^{-15}\text{ eV}\cdot\text{s}$ $\quad \hbar = 1.05\times10^{-34}\text{ J}\cdot\text{s} = 6.585\times10^{-16}\text{ eV}\cdot\text{s}$

Integral table provided on separate sheet.

1. (8 points) Circle either **T**(rue) or **F**(alse) for the following statements about a material in its superconducting state:

T F The electric field is zero within the superconductor.

Magnetic field zero; E field zero in conductor in static equilibrium only

T F If the current flowing through the superconductor is too low, it loses its superconducting properties.

Can quench if current too high

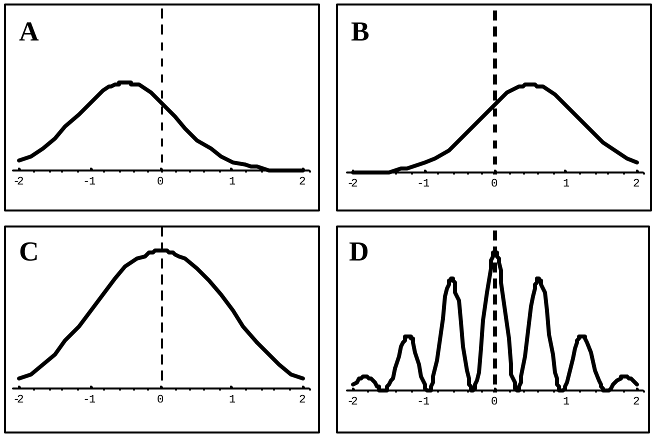
T F If it is heated up enough, it loses its superconducting properties.

Can quench if $T > T_c$

T F The superconducting state relies on the fact that electrons are bosons.

Electrons are fermions; they Cooper pair up to form bosons

2. (12 points) Consider two narrow, closely spaced slits. Below are four possible intensity (vertical axis) vs. position (horizontal axis) patterns that could be formed on some kind of screen. Unless otherwise indicated, assume there is no detector at either slit. For each situation below, write down as many letters correspond to that situation as necessary.



a) Pattern formed by electrons going through the slits, one electron at a time (assume electrons have same velocity).

D. Electrons have wave characteristics; single electrons show interference pattern

b) Pattern formed by classical particles going through the slits.

C. Classical particles just add.

c) Pattern formed when one of the slits is closed off.

A & B. Correspond to single slit diffraction.

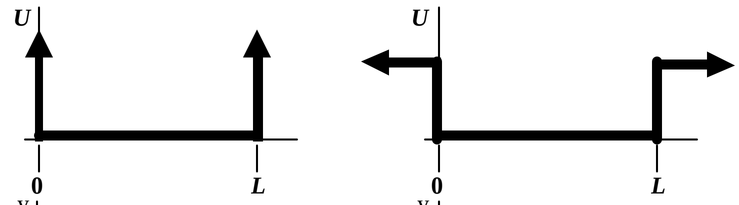
d) Pattern formed by a laser beam incident on the slits.

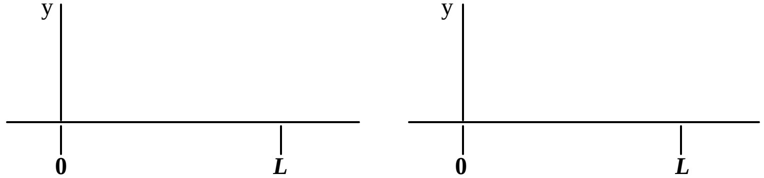
D. Photons have wave characteristics; show interference pattern

e) Pattern formed by beam of coherent electrons going through the slits, with an electron detector at ONE of the slits.

C. If know what slit electrons pass through, electrons are forced to act like particles (localized). So get pattern associated with classical particles.

3. (20 points) Consider an infinite square well potential and a finite square well potential, both with the same width L as shown below.





a) On the axes provided below the **infinite square well**, carefully sketch the wave function of a particle of mass *m* in its first excited state (i.e. the second lowest energy state).

b) On the axes provided below the **finite square well**, carefully sketch the wave function of a particle, also of mass *m*, in its first excited state (i.e. the second lowest energy state).

c) Carefully indicate on your sketch of the wave function for the **finite square well** the region(s) where you are *most likely* to find the particle.

Indicated by arrows.

d) Call the energy of the state you sketched for the infinite square well E_a. Call the energy of the state you sketched for the finite square well E_b. Which of the following is true about the relationship between E_a and E_b? (Circle one.)

$E_a > E_b$ $\qquad$ $E_a = E_b$ $\qquad$ $E_a < E_b$ $\qquad$ not enough information to compare energies

From sketch, can see that inside wells, wavelength in finite square well is longer than wavelength in infinite square well: $\lambda_a < \lambda_b$**. From de Broglie,** $p = h/\lambda \Rightarrow p_a > p_b$**. In well, energy is just kinetic energy** $E = K + U = K = p^2/2m$**, so** $E_a > E_b$.

4. (17 points) Consider a hydrogen atom that has been excited to some unknown state.

a) You measure the z-component of orbital angular momentum, and find it to be $-2\hbar$. Write down a possible combination of n, ℓ, and m_ℓ that corresponds to the LOWEST energy state this hydrogen atom could be in.

z-component of spin: $L_z = m\hbar = -2\hbar \Rightarrow m = -2$

$m = -\ell, -\ell+1, \ldots, \ell-1, \ell+1 \Rightarrow \ell = 2, 3, \ldots$ **since** $m = -2$

$\ell = 0, 1, 2, \ldots, n-1 \Rightarrow n = 3, 4, \ldots$ **since** $\ell = 2, 3, \ldots$

Lowest energy state means that $n = 3$**, since** $E_n = \dfrac{-13.6}{n^2}$**eV for hydrogen. So lowest state must be**

$$n = 3, \ell = 2, m = -2,$$

since told that $m = -2$**.**

b) The atom eventually transitions down to its ground state, with normalized wave function $\psi = \dfrac{1}{\sqrt{\pi}}\left(\dfrac{1}{a_0}\right)^{3/2} e^{-r/a_0}$, where a_0 is the Bohr radius. Determine the probability of finding the electron in the region $2a_0 < r < \infty$ Show all your work for full credit.

For hydrogen in ground state, $P(r) = 4\pi r^2 \psi^2$**, since only function of** r**. So** $P = \int_{2a_0}^{\infty} P(r)dr = \int_{2a_0}^{\infty} 4\pi r^2 \psi^2 dr$

$$= 4\pi \int_{2a_0}^{\infty} r^2 \left(\frac{1}{\sqrt{\pi}}\left(\frac{1}{a_0}\right)^{3/2} e^{-r/a_0}\right)^2 dr = \frac{4\pi}{\pi}\frac{1}{a_0^3}\int_{2a_0}^{\infty} r^2 e^{-2r/a_0} dr$$

$$= \frac{4}{a_0^3}\int_{2a_0}^{\infty} r^2 e^{-2r/a_0} dr = \frac{4}{a_0^3}\left(-e^{-2r/a_0}\right)\left[\frac{r^2}{2/a_0} + \frac{2r}{(2/a_0)^2} + \frac{2}{(2/a_0)^3}\right]_{2a_0}^{\infty}$$

$$= \frac{4}{a_0^3}\left(-e^{-2\infty/a_0}\right)\left[\frac{\infty^2}{2/a_0} + \frac{2\infty}{(2/a_0)^2} + \frac{2}{(2/a_0)^3}\right]$$

$$- \frac{4}{a_0^3}\left(-e^{-2(2a_0)/a_0}\right)\left[\frac{(2a_0)^2}{2/a_0} + \frac{2(2a_0)}{(2/a_0)^2} + \frac{2}{(2/a_0)^3}\right] =$$

$$= 0 - \frac{-4e^{-4}}{a_0^3}\left[2a_0^3 + 1a_0^3 + \frac{1}{4}a_0^3\right] = 4e^{-4}[3.25] = 0.238$$

5. (22 points) Many protons are prepared in the state $|\psi\rangle = -a|+z\rangle + 2a|-z\rangle$, where a is a constant, $|+z\rangle$ means the z-component of spin is positive, and $|-z\rangle$ means the z-component of spin is negative. These protons are in a uniform magnetic field $\vec{B} = 3.4\hat{k}$, where the magnetic field is measured in Tesla.

a) You measure the z-component of the spin angular momentum of this proton. Determine the **numerical probability** of finding the z-component of spin to be $-\hbar/2$.

Probability of finding in particular state is coefficient for that state squared, so probability of finding $-\hbar/2$ probability of finding in $|-z\rangle$, so $\Rightarrow (2a)^2 = 4a^2$. Need numerical value for a. State must be normalized, so sum of coefficients square equals 1:

$$(-a)^2 + (2a)^2 = 1 \Rightarrow a^2 + 4a^2 = 5a^2 = 1 \Rightarrow a^2 = \frac{1}{5}.$$

So $4a^2 = 4\frac{1}{5} = \frac{4}{5}$

b) Let's say that you find one of the protons to have z-component of spin angular momentum $-\hbar/2$. What wavelength photon is associated with a spin-flip transition of this proton?

Energy of spin in magnetic field given by $E = U = \pm\mu_z B$, depending on if spin is parallel or anti-parallel to field. So spin-flip transition energy $\Delta E = 2\mu_z B$. Photon has energy of transition: $E_\gamma = \frac{hc}{\lambda} = \Delta E = 2\mu_z B \Rightarrow \lambda = \frac{hc}{2\mu_z B} =$

(1240 eV nm) / (2 x 8.8 x 10^{-8} eV / T x 3.4 T)
= 2.07 x 10^{9} nm = 2.07 m.

5c) Again, many protons are prepared in the state $|\psi\rangle = -a|+z\rangle + 2a|-z\rangle$, and are placed in the magnetic field $\vec{B} = 3.4\hat{k}$ T. Determine the expectation value of the **energy** of these protons.

Expectation value of energy:

Sum of prob. of being in state x value of energy for state. Spin in fields have energy $E = U = \pm\mu_z B$, sign depends on if spin component parallel or anti-parallel to field;

$|+z\rangle$: **parallel to field:** $E = -\mu_z B = -3\times 10^{-7}$ eV;

probability: $a^2 = \frac{1}{5}$

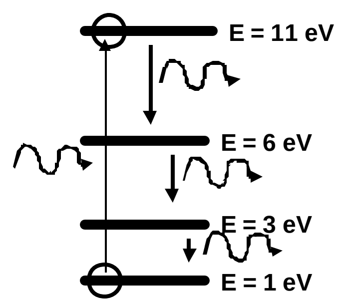
$|-z\rangle$: **anti-parallel. to field:** $E = +\mu_z B = +3\times 10^{-7}$ eV;

probability: $4a^2 = \frac{4}{5}$.

$$\langle E\rangle = \frac{1}{5}(-3\times 10^{-7}) + \frac{4}{5}(3\times 10^{-7}) = \frac{3}{5}(3\times 10^{-7}) = 1.8\times 10^{-7}$$
$$= 1.8\times 10^{-7} \text{ eV}.$$

6. (6 points) When you crush a Wint-O-Green Lifesavers™ candy between your teeth, you disrupt the crystal structure of the candy. Photons of energy 10 eV (approximately) are emitted from processes associated with this disruption; however these photons are outside the visible spectrum. Briefly **explain** why you are able to see sparks when you bite into a Wint-O-Green candy (don't just name a phenomenon).

Some substance in the candy absorbs the 10 eV photons, going from its ground state to some high excited state. This substance then transitions down from its excited state to its ground state in "steps", each transition to a lower energy state emits a photon; some of these photons are visible. An example scheme is shown.



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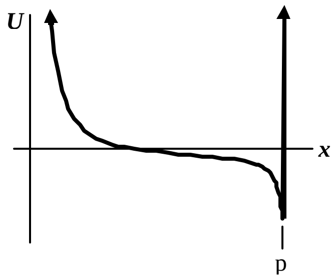
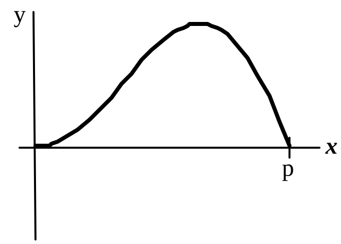
7. (15 points) A particle of mass m is in a peculiar potential given by $U(x)=U_0\dfrac{\cos(x)}{x\sin(x)}$ (wit U_0 constant) in the region $0\le x\le\pi$, and $U(x)=\infty$ elsewhere. This potential is sketched to the right.



A particle with total energy E is **known** to have the wave function $\psi(x)=Ax\sin(x)$ in the region $0\le x\le\pi$, and $\psi(x)=0$ elsewhere; this wave function is also sketched to the right.



Remember that the Schroedinger Equation is:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2}+U(x)\psi(x)=E\psi(x).$$

Note that $U(x)\psi(x)=U_0\dfrac{\cos(x)}{x\sin(x)}Ax\sin(x)=U_0A\cos(x)$ in the region $0\le x\le\pi$.

Determine the energy E of the particle as well as the constant U_0. Leave your answer in terms of fundamental constants and the given m.

Obtain second derivative of wave function. Substitute into S.E. Group by like functions of x.

$$\psi(x)=Ax\sin(x)$$
$$\frac{d\psi(x)}{dx}=A\sin(x)+Ax\cos(x)$$
$$\frac{d^2\psi(x)}{dx^2}=A\cos(x)+A\cos(x)-Ax\sin(x)$$
$$=2A\cos(x)-Ax\sin(x)$$

7. (continued)

Now, substitute into Schroedinger Equation:

$$-\frac{\hbar^2}{2m}(2A\cos(x)-Ax\sin(x))+U_0A\cos(x)=EAx\sin(x)$$

$$-\frac{\hbar^2}{2m}2\cos(x)+\frac{\hbar^2}{2m}x\sin(x)+U_0\cos(x)=Ex\sin(x)$$

Group by like x terms.

$$\left(\frac{\hbar^2}{2m}-E\right)x\sin(x)+\left(U_0-\frac{\hbar^2}{m}\right)\cos(x)=0$$

This must be true for all values of x, so try:

$x=0$:

$$\left(\frac{\hbar^2}{2m}-E\right)(0)\sin(0)+\left(U_0-\frac{\hbar^2}{m}\right)\cos(0)=U_0-\frac{\hbar^2}{m}=0$$

$$\Rightarrow U_0=\frac{\hbar^2}{m}$$

and

$x=\dfrac{\pi}{2}$:

$$\left(\frac{\hbar^2}{2m}-E\right)\left(\frac{\pi}{2}\right)\sin\left(\frac{\pi}{2}\right)+\left(U_0-\frac{\hbar^2}{2m}\right)\cos\left(\frac{\pi}{2}\right)=\left(\frac{\hbar^2}{2m}-E\right)\left(\frac{\pi}{2}\right)=0$$

$$\Rightarrow\frac{\hbar^2}{2m}=E$$

or just note that coefficients of each x term must be zero.