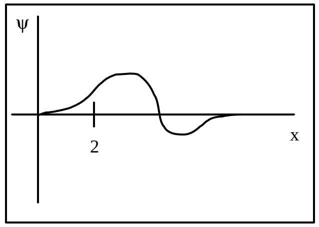
Physics 212E – Chowdary
Classical and Modern Physics
Spring 2005

PX-4
(Practice Exercises Set 4)

1) An electron has a wavefunction as shown in the sketch to the right. In the region $0 < x < 2$, the wavefunction is described by the function $\psi(x)=0.2x^2$, and $\psi(x) = 0$ for $x < 0$.



a) Calculate the probability that a measurement would locate the particle in the region $0 < x < 2$.

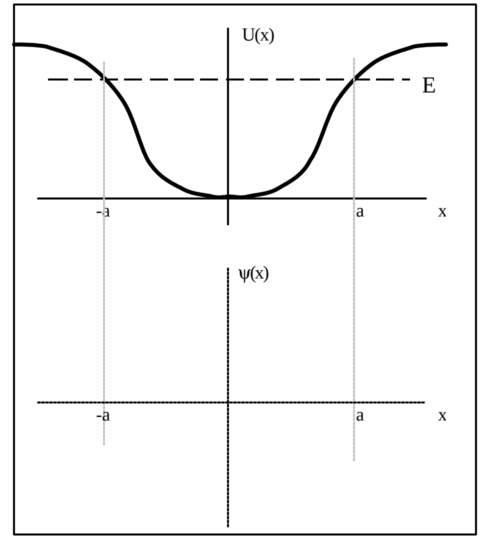
b) Calculate the probability that the particle will be found in the region $x > 2$.

2) An electron is trapped in an infinite potential well (i.e., a box) with width 2.3 nm. It is in a particular state with well-defined energy. The state is such that there are two places within the box (4 if you include the edges) where you would never be able to find the particle.

a) Calculate the kinetic energy of the electron.

b) The electron makes a transition to its lowest energy state. Determine the wavelength of the photon associated with this transition, and whether this photon is absorbed or emitted.

3) The figure shows a plot of potential energy vs. position, together with a constant E-value for the 3rd excited state (i.e., the 4th lowest energy state) of the system. On the lower axes, carefully sketch the wavefunction for this state.



4) The time independent Schroedinger equation for a particle of mass m in a harmonic oscillator potential is $-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2}+\frac{1}{2}m\omega_0^2x^2\psi(x)=E\psi(x)$, where ω_0 is the natural frequency of the oscillator. Verify that $y(x) = Ae^{-bx^2}$ is a solution to the Schroedinger equation, and solve for b and E in terms of fundamental constants and the givens m and ω_0.

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PX-5
(Practice Exercises Set 5)

1) The electron spin states $|+z\rangle$ and $|-z\rangle$, which have z-components of spin equal to $+\frac{1}{2}\hbar$ and $-\frac{1}{2}\hbar$ respectively, can be written in terms of states with definite x-component of spin as follows:

$$|+z\rangle = \frac{1}{\sqrt{2}}|+x\rangle + \frac{1}{\sqrt{2}}|-x\rangle \qquad |-z\rangle = \frac{1}{\sqrt{2}}|+x\rangle - \frac{1}{\sqrt{2}}|-x\rangle$$

Given an electron in the spin state $|\psi\rangle = -0.28|+z\rangle + 0.96|-z\rangle$ determine the probability that a measurement of the *x-component* of spin will result in the value $+\frac{1}{2}\hbar$.

2) A one dimensional potential well has width L = 1.00 nm, and contains 5 electrons (for this problem, you may neglect the energy associated with the repulsive interaction between electrons.) The system begins in its ground state, the state with the lowest possible total energy. The system absorbs a photon. Determine the longest possible wavelength this photon can have.

3) An electron in a hydrogen atom is in the state

$$|\psi\rangle = 0.5|1\rangle + 0.7|2\rangle + c|3\rangle$$

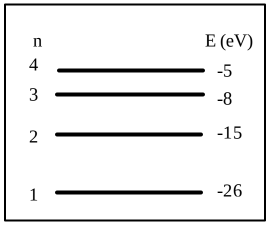
where state $|1\rangle$ corresponds to the ground state with energy E_1, $|2\rangle$ the state with energy E_2, etc.

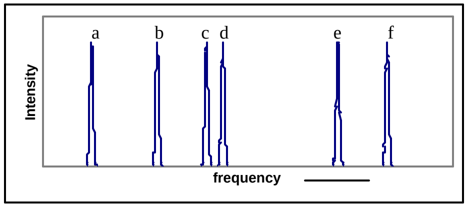
a) Determine a value for the coefficient c.

b) Determine the expectation value for the energy $\langle E\rangle$. Your result should be a numerical value in units of eV.

c) What is the most probable result for an energy measurement of this electron? Your result should be a numerical value in units of eV.

4) An atom has the four energy levels as shown in the figure below. The spectrum of this atom is also shown, and contains 6 peaks.





a) Determine the atomic transition responsible for each peak. (Fill in the table to the right.)

b) Determine the wavelength of the light emitted when the electron undergoes a transition from the n=4 to the n=2 level.

Peak in the spectrum	Initial n (before transition)	Final n (after transition)
a		
b		
c		
d		
e		
f		

PX-6
(Practice Exercises Set 6)

1) The wave function of a hydrogen atom in one of its excited states is given by $\psi_{210} = C\frac{r}{a_0}e^{-r/2a_0}\cos\theta$, where C is the normalization constant and a_0 is the Bohr radius.

a) You measure the energy of this electron. What are the possible results of this measurement?

b) You measure the z-component of the spin magnetic moment of this electron. What are the possible results of this measurement?

c) You measure the position of this electron. What is the probability of finding the electron in the first octant? You can leave your answer in terms of given and fundamental constants.

2) The ground state of a hydrogen atom has energy E_1 = -13.6eV. An electron is in the 4th excited state (n = 5).

a) List the possible values for the orbital quantum number ℓ for this electron.

b) The electron now drops to the n = 4 state. Find the wavelength of the photon emitted in this process.

c) While in the n = 4 state, you measure the z–component of orbital angular momentum. What is a possible result of this measurement?

3) A patient undergoes an MRI scan to detect a possible brain tumor. Assume the patient's brain is in spatially varying magnetic field described by $B(x) = 1.100 + 2.200x$, where B is in Tesla and x is in meters. At what location in the brain would you expect to detect a resonant absorption, if the incoming photons have frequency 50 MHz? Assume that all the absorption is due to proton spin flips and neglect internal magnetic fields.

4) A cylinder with a temperature of 40 K is in an external magnetic field B_{ext} = 0.23 T. The temperature is now dropped to 20 K, which is below the cylinder's critical superconducting temperature of 30 K. Determine the total magnetic field inside the cylinder at the end of this process.