E Problems for Unit III

E48: Classical Uncertainty Principle. In this activity, you will use your Magic Spring to explore the classical uncertainty principle.

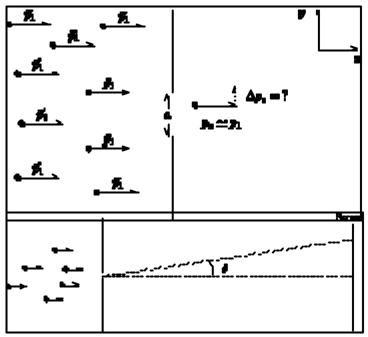
a) Attach one end of the spring to a fixed object or partner and start a single sharp pulse that travels along the spring. At some instant, estimate the wave's location and wavelength. One of these should be easy to estimate, and one should be difficult; which is which?

b) Now, shake the end up and down periodically so that many cycles travel down the spring. Again, estimate the wave's location and wavelength. Which was easier to estimate?

c) How does this classical wave behavior connect to the quantum uncertainty principle?

E49: Uncertainty and Diffraction:

Particles with mass m and horizontal momentum $\vec{p}_1$ are formed into a wide beam. The beam is sent towards a barrier with a vertical slit opening of width a. Consider one particle which you know makes it through the slit, as shown in the figure. You know essentially nothing else about this particle except that its mass hasn't changed.



a) What is the approximate uncertainty in the vertical position of the particle immediately after it passes through the slit?

b) Determine the approximate minimum uncertainty in the vertical momentum of the particle right after it passes through the slit.

c) The particles are detected on a screen placed far from the slit. Show that you expect to find the particles hit the screen over a range of angles $\pm\theta$, where $\tan\theta \approx \frac{\lambda}{2\pi a}$.

Note that you may get $\frac{1}{2}\frac{\lambda}{2\pi a}$ or even $\frac{1}{4}\frac{\lambda}{2\pi a}$ instead.

E50: Life in a Quantum World. Last semester we asked you to describe some relativistic effects that you would notice if the physical "speed limit" of light were 50 mph instead of 3×10^8 m/s. Now imagine that the quantum constant $\hbar$ were 1 Joule-sec instead of 1.05×10^{-34} Joule-sec. Note that [J][s]=[kg m²/s²][s] = [m][kg m/s]

a) The uncertainty principle in this world would be $\Delta x\Delta p \geq \frac{1}{2}$ Joule-sec. Imagine that someone throws a ball to you. Describe what you would experience trying to catch the ball.

b) Consider the energy of a photon. With visible light frequencies still in the 10^{14} Hz range, describe how it would feel to be sunbathing on a beach.

c) What would be your approximate de Broglie wavelength when walking (use estimates of your mass and typical walking speed)? What would happen when you walk through a doorway?

E51: Localization. Obtain the file "localization.xls". Go to "My Computer" on your desktop. Go to "departments on netspace". Find the physics department folder, and go into the public folder. From there, find the PHYS212E folder, which is where you'll find localization.xls. Open up the worksheet and immediately "Save As" to your own network space. The worksheet is "protected" so that you don't accidentally change any important formulas, but that also hides those formulas. If you want to see the formulas or manipulate them, simply go to Tools → Protection, and then Unprotect Sheet… If you want to print out any of the graphs you make, you should just print the first page of the worksheet.

This worksheet will allow you to plot the superposition of the various energy (normal) modes for the particle in the box, up to n = 20. You should see a series of yellow cells, most of which should be entered as zero. The cell immediately under n = 1 should be entered as 1, so that initially only the first mode is non-zero. There should be three graphs displayed in the worksheet. The top graph should show the superposition of the wave functions y for any modes with amplitude not equal to zero. The middle graph should show |y|² (i.e. the probability distribution) for this superposition state. The bottom graph should show the individual wave functions for any non-zero modes. Note that the superposition state is normalized.

a) With A_n = 1 for the n = 1 mode, the graphs of the wave function and the probability distribution should look very familiar. Change the amplitude for the n = 2 mode to 1 also. What do you see? Does this pattern make sense to you?

b) What do you think will happen to the pattern if you "turn on" the first five modes, from A_n = 0 to A_n = 1? Write down your prediction.

c) Now, test your prediction from part b) by changing the amplitudes of the first five modes so that they are all 1. What do you see? Was your prediction correct? Does this shape make sense to you?

d) Next, "turn on" all 20 modes, with A_n = 1. Describe what you see. Note that adding higher order energy modes is really increasing the spread in momentum present in the box. Does the behavior you observe qualitatively match what you would expect, given the Uncertainty Principle?

e) You've just observed that adding together multiple energy modes can localize the particle in the box. What is the effect of adjusting amplitude? In the previous steps, you set all the A_n's = 1, but the spreadsheet allows you to set them to any value you choose. Play around with the relative amplitudes, and comment on the role the amplitudes play in the localization process.

E52: Parking Lot Probabilities. In this problem, you'll explore the idea of a probability distribution, classically. Clark Kent, an aspiring physics student, works part-time parking cars at a downtown hotel. The lot is a long underground tunnel, with all the cars parked in a single long row, 600 m long. When owners return for their cars, instead of telling them where to find their car, he describes the location in terms of probability and probability density.

a) Lana Lang is told that her car "could be anywhere in the lot", which means that the probability density is constant. Calculate the value of this uniform probability density $P(x)$ for Lana to find her car a distance x from one end of the lot. (Answer in units of probability / m.)

b) Find the probability that Lana's car is in the first 100m.

c) Lois Lane is told that the probability density to find her car is a constant P_1 in the first third of the lot, and a second constant $P_2 = 1/3\,P_1$ in the last two-thirds of the lot. Find the different constant probability densities P_1 for $0 < x < 200$ m and P_2 for $200\text{ m} < x < 600$ m.

d) Determine the probability that Lois's car is in the first 400 m of the parking lot.

E53: Electron-in-the-Box. An electron is confined to a one-dimensional infinite potential well, and is in the 3rd mode (second excited state).

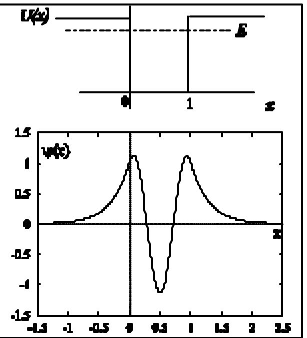
a) Draw a sketch of the wave function ψ corresponding to this mode, and indicate the locations where you would never expect to find the electron, and where you are most likely to find the electron.

b) Determine the wavelength of the electron, if it has an energy of 75 eV.

c) Using your answers to parts a) and b), determine the width of the well.

E54: Finite Square Well Probabilities. Consider a particle with mass m in the illustrated one-dimensional *finite* square well. The normalized wave function for this particle is provided below, along with a sketch (you can assume that distances are in meters). We'll study the finite well and its amazing consequences during class on Friday, April 1, but you should already be able to do lots of things with this problem.

$$\psi(x) = \begin{cases} 1.02e^{3.48x} & \text{for } x \le 0 \\ -1.13\cos(7.20x - 3.60) & \text{for } 0 < x < 1 \\ 33.04e^{-3.48x} & \text{for } x \ge 1 \end{cases}$$



a) You may have noticed that the total (mechanical) energy of the particle in the well is less than the height of the well. Since $E = K + U$, determine the sign of K in the region inside the well where $U(x) = 0$ and in the region outside of the well, where $U(x) > E$. One of these answers should bother you. Which one, and why?

b) The position of the particle is measured. Set up and evaluate the integral that will allow you to calculate the probability that the particle will be found to the left of the origin, that is, in the region $x \le 0$.

c) Set up (but **do not evaluate**) the integral that would allow you to calculate the probability of finding the particle to the right of the well, that is, in the region $x \ge 1$.

d) Use a symmetry argument to determine the probability of finding the particle to the right of the well.

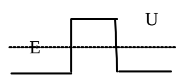
e) Set up (but **do not evaluate**) the integral that would allow you to calculate the probability of finding the particle inside the well, that is, in the region $0 < x < 1$.

f) **Without evaluating the integral** from part e), determine the probability of finding the particle inside the well.

E55: Aluminum Nano-Layer. An aluminum layer of thickness 1.0 x 10^{-7} m is deposited on a microelectronics chip. The aluminum is a very good conductor, so electrons can move freely in it. The aluminum layer may therefore be modeled as an infinite potential well of width 10^{-7} m. What is the smallest frequency of light that can be emitted from this system?

E56: Schroedinger Equation Worksheet. Please work through the *Schroedinger Equation Worksheet* handed out in class on Friday, April 1.

E57: Barrier Tunneling. Consider a particle approaching from the left and trying to "tunnel through" a classically forbidden barrier region as shown.

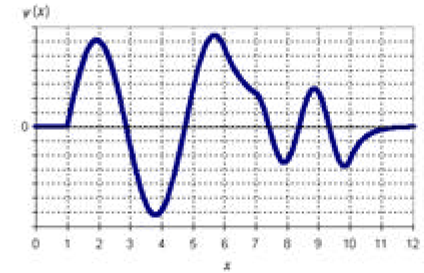


a) Sketch the wave function of the particle to the left of the barrier.

b) Sketch the wave function inside the barrier.

c) Describe briefly how a quantum particle can get through this classically forbidden barrier.

E58: Wave functions and Potential Energy. Consider the following sketch of the wave function of a particle in some piecewise constant potential. Your goal is to make a qualitative sketch of the potential energy $U(x)$ versus x associated with this wave function.



a) Begin by assuming that the total energy E of the particle is constant. Convince yourself that the wave function is different in the following regions: $x < 1$; $1 < x < 6$; $6 < x < 7$; $7 < x < 10$; and $x > 10$. Briefly describe what changes from region to region.

b) Look for any regions where the wave function is constantly zero. What does that mean about the probability of finding the particle in those regions? What does that tell you about the potential energy in those regions?

c) Look for any regions where the wave function is oscillatory. What does this tell you about the relationship between E and U in those regions (i.e. which one is larger)?

d) Continue to look at regions where the wave function is oscillatory. Compare the wavelengths in those regions. What does a larger wavelength tell you about momentum? What does that tell you about kinetic energy? What does kinetic energy tell you about the comparison between E and U?

e) Look for any regions where the wave function is exponential. What does this tell you about the relationship between E and U in those regions?

f) Now put all these pieces together and sketch the potential energy as a function of position.

E59: Double-slit with Electrons. Consider a double-slit experimental set-up, with two closely spaced narrow slits. A beam of electrons with the same kinetic energy is incident on the slits. On the other side of the slits is a screen that can record or show the pattern of electrons (for example photographic film or some kind of phosphor screen such as in a television.)

a) Sketch (or somehow represent) the pattern on the screen if only ONE of the slits were open. Also, sketch (or somehow represent) the pattern on the screen if only the OTHER slit were open.

b) Sketch (or somehow represent) the pattern on the screen if BOTH slits were open simultaneously.

c) Describe how the patterns you drew in parts a), b) and c) would change if you decreased the intensity of the electron so that only one electron at a time was emitted. The emission rate could be determined by counting the number of spots that appear on the screen at one time. Assume that you allow this process to happen long enough that a significant number of electrons hit the screen.

d) How would the patterns you drew in parts a), b) and c) change if you turned on a detector that determined which slit the electron went through?

e) How would this experiment change if you used a beam of photons with the same wavelength instead of a beam of electrons?

E60: Proton-in-the-Box. A proton is confined to a one-dimensional infinite potential well with a width of 2 x 10^{-14} m. Assume that it is in its first excited state (i.e., not the ground state, but the next mode).

a) Draw a sketch of the wave function ψ corresponding to this mode, and indicate the regions where you would never expect to find the proton.

b) Determine the wavelength of the proton wave.

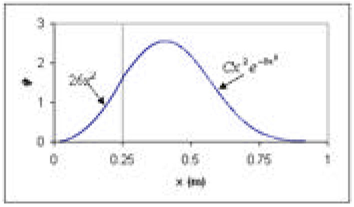
c) Using your answer to part b), determine the magnitudes of the momentum and energy of the proton in the first excited state.

d) The proton makes a transition from the first excited state to the third excited state. Determine the wavelength of the photon that could cause this transition. Is this photon absorbed or emitted?

361: Wave function and Probability. The wave function of a particle is

$\psi(x) = 0$ for $x \le 0$, $\psi(x) = 26x^2$ for $0 \le x \le 0.25$ m,

and $\psi(x) = Cx^2 e^{-bx^3}$ for $x \ge 0.25$ m, where C and b are real constants, as sketched. Determine a numerical value for the probability of finding the particle in the region $x \ge 0.25$ m.



362: Transitions in Box. A particle confined to a one-dimensional box has a ground state energy of 0.4 eV. When irradiated with light of 206.7 nm it makes a transition to an excited state. When decaying from this excited state to the next lower state, it emits radiation of 442.9 nm. What is the quantum number of the state to which the particle has decayed?

363: Particle-in-a-2D-Box. A particle of mass m in a two-dimensional box of width L_1 and length $L_2 = 2L_1$ has wave function $\psi_{n_1 n_2}(x,y) = A\sin\left(\frac{n_1 \pi x}{L_1}\right)\sin\left(\frac{n_2 \pi y}{L_2}\right)$

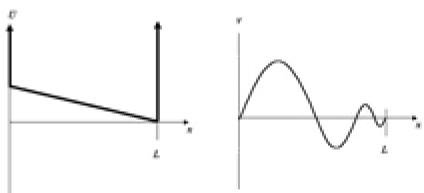
with energy $E_{n_1 n_2} = \frac{\hbar^2 \pi^2}{2m}\left[\left(\frac{n_1}{L_1}\right)^2 + \left(\frac{n_2}{L_2}\right)^2\right] = E_0\left(4n_1^2 + n_2^2\right)$.

a) Determine E_0 in terms of fundamental constants and givens.

b) List *and identify* the quantum numbers (n_1, n_2) for the lowest energy states. Give the energies of these 6 states as a multiple of E_0.

c) Of those six lowest states, which if any are degenerate?

364: Shapes of Wells and Wave Functions. Consider the potential well shown below to the side. A wave function for a particle trapped in this potential is shown below on the right.

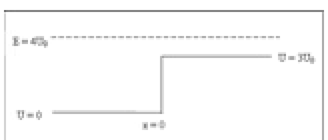


a) Explain how you know the particle is in its third excited state.

b) Explain why the wave function goes to zero at the end points.

c) Explain why the wavelength varies as it does.

365: Step Potential. A particle of mass m and total energy $E = 4U_0$ approaches the step barrier shown below. In the region $x < 0$, the potential energy $U = 0$. In the region $x > 0$, the potential energy $U = 3U_0$.



a) In what region ($x < 0$ or $x > 0$) does the particle have the larger kinetic energy? The larger momentum? The larger wavelength?

b) Determine the wave number k (in terms of m, fundamental constants, and quantities in the sketch) in the region $x < 0$.

c) The particle starts from the far right and moves towards the left. Calculate the probability that the particle will be reflected at the step.

366: Testing Schroedinger's Equation. The time independent Schroedinger equation for a particle of mass m confined to a one-dimensional box is $-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} = E\psi(x)$.

a) Does the wavefunction $\psi(x) = A\cos(kx)$ (with A and k real constants) satisfy the Schroedinger equation? Show your work in answering this, and if it does satisfy the Schroedinger equation, then determine an expression that relates E to constants in the Schroedinger equation and in the given wavefunction.

b) Does the wavefunction $\psi(x) = Be^{\kappa x}$ (with B and κ real constant) satisfy the Schroedinger equation? Show your work in answering this, and if it does satisfy the Schroedinger equation, then determine an expression that relates E to constants in the Schroedinger equation and in the given wavefunction.

367: Expectation Values for Classical Dice. In a fair six-sided die, the probability of rolling any number between 1 and 6 should be 1/6. Consider an experiment where you roll a die and add up the numbers that come up in each roll.

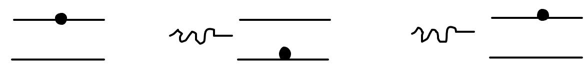
a) If you roll the die 600 times, how many times do you expect the number 3 to occur? If you add up all these 3's, what will you obtain? Repeat part for rolls where 1, 2, 4, 5, and 6 occur. What total number should you arrive at?

b) Calculate the average number you rolled in the regular fashion, by taking the total sum of your individual dice rolls and dividing by 600 (the total number of times you rolled the die).

c) Determine the expectation value (recall that expectation value is just the average value) using the ideas behind Eq. (1.11) in Supplemental Chapter 1.

d) Consider the expectation value you obtained in parts b) and c) (which should be the same). Would you ever have gotten this value in any single measurement (roll of the die)? Does this bother you?

368: Photon-Atom Processes. The sketches below show the state of a two-level atom and possibly a photon. For each "Before Sketch" make a corresponding "After Sketch" and name the process.



369: Continuous vs. Discrete Spectra. Take your diffraction glasses and use them to look at as many of these different light sources as you can find: street lamps, stadium lamps, car head lights (DON'T DO THIS WHILE DRIVING!), digital clock faces, tv screens, flat panel monitors (there are plenty in the library), fluorescent lights, and neon lights (if you can, check out the marquis at the Campus Theater downtown). It is best if the light source is far away and as point-like as you can make it. Which of sources show continuous spectra, and which (if any show) discrete lines? Explain how a source could produce discrete lines.

370: Superposition-in-the-Box. A one-dimensional box has a length such that a particle trapped in that box can have energies $E_n = n^2 E_1$, where $E_1 = 2.0$ eV and n = 1, 2, 3, … Many identical boxes, each containing a single particle, are all prepared in the same superposition state: $|\psi\rangle = 0.8|1\rangle + b|2\rangle + 0.4|3\rangle + 0.2|4\rangle$, where $|n\rangle$ is the state corresponding to the energy E_n.

a) Assuming that the state $|\psi\rangle$ is normalized, determine a possible value for the coefficient b.

b) What is the expectation value for the energy of a large number of particles prepared in this state?

c) You now measure the energy of one of the particles. What is the probability that you will find the energy to be 18 eV?

d) Let's say that you *do* find an energy of 18 eV for the particle. What can you say about the energy of the particle before the measurement was made and immediately after the measurement was made?

371: Population Inversion & Lasers. Describe "population inversion." Explain briefly why a population inversion is necessary for the operation of a laser.

72: **Self-Fluorescence.** You each should have several pieces of Wint-O-Green Life Savers. The chemical methyl salicylate (also known as the oil of wintergreen) is the fluorescing chemical in this candy. I want you to see if you can observe the fluorescence yourself. It is absolutely imperative that you stand in a completely dark room/closet/bathroom, etc. Block the light that comes in from outside with a towel, etc. Wait a few minutes for your eyes to adjust. If you can convince a friend to come with you, you can try to crack the Life Saver in your mouth (you can chew with your mouth wide open this one time; you have permission from your physics professor) and your companion can attempt to observe sparks. I also was successful with crushing the Life Saver between two spoons, as well as observing myself with a mirror, since I was trying this by myself.

a) Describe your attempt. Where were you, who were you with, what did you do, and did you see sparks? What colors were the sparks? Estimate the wavelength, frequency, and energy (both in Joules and eV) of the sparks you saw / might see.

b) Describe fluorescence in a way that someone outside of our class might understand. By this, I mean try to use everyday non-technical language, and try to make analogies to situations that a person not studying quantum mechanics would understand.

73: **Recoil Emission.** The electron in a hydrogen atom is in the n = 2 state. The electron makes a transition to the ground state.

a) What is the energy of the photon according to the Bohr model?

b) The linear momentum of the emitted photon is related to its energy by $p = E/c$. If we assume conservation of linear momentum, what is the recoil velocity of the atom?

c) Find the recoil kinetic energy of the atom in electron volts. By what percentage must the energy of the photon in part (a) be corrected to account for this recoil energy?

74: **Spin-flip Photon.** What is the wavelength of a photon that will induce a transition of an electron spin from parallel to anti-parallel orientation in a magnetic field of magnitude 0.200 T?

75: **Spin-flip Field.** An external oscillating magnetic field of frequency 34 MHz is applied to a sample that contains hydrogen atoms. Resonance is observed when the strength of the constant external magnetic field equals 0.78 T. Calculate the strength of the local magnetic field at the site of the protons that are undergoing spin flips, assuming the external and local fields are parallel there.

76: **Insane in the Brain?** Assume that the magnetic field along a line passing through a patient's brain in an MRI scan is described by the function $B(x) = 0.5 + 0.6x$, where B is in Tesla and x is in meters.

a) What is the location in the brain where protons will flip in response to a 30 MHz oscillating magnetic field?

b) If you want to probe a possible tumor at a position of $x = 0.50$ m, at what frequency should you oscillate the magnetic field?

c) For your answer in part b, what is the energy of the photons that are probing your patient (in eV)? Considering that the weakest molecular bonding energies are around 0.1 eV, is this safe for your patient?

(Note: this is, of course, a simplification of what is really happening in an MRI. The actual problem is complicated by the fact that the local magnetic field at the proton to be flipped is not equal to the imposed external magnetic field, and, in fact, this local magnetic field depends on the surrounding material.)

77: **Spin Energy and Expectation Value.** Many protons are prepared in the state $|\psi\rangle = -\sqrt{\frac{1}{3}}|+z\rangle + \sqrt{\frac{2}{3}}|-z\rangle$, where $|+z\rangle$ means the z-component of spin is positive, and $|-z\rangle$ means the z-component of spin is negative. These protons are in a uniform magnetic field $\vec{B} = 3.4\hat{k}$, where the magnetic field is measured in Tesla.

a) You make a measurement on one of the protons, and determine that the z-component of the spin is positive, in other words that it is aligned with the magnetic field. Is this the low energy or the high energy state? Calculate a numerical value for the energy of this proton.

b) Calculate a numerical value for the expectation value of the energy for the state $|\psi\rangle$.

78: **How Cool is That!** In problem session, you will observed a ceramic disk reduced below its superconducting temperature by immersion in liquid nitrogen. This problem is in conjunction with Supplementary Reading Chap. 3: 3.2 and 3.3.

a) Closely observe the little cube hovering over the disk. Comment on what you observe. What evidence do you have that this is a superconductor? Can you make the cube spin?

b) Explain how the superconductor can levitate the magnet.

79: **Differentiating Radial Probability Distribution.** The radial probability distribution function of the electron in hydrogen in its ground state is given by $P(r) = Cr^2 e^{-2r/a_0}$, where C is some constant, and a_0 is the Bohr radius. Take the derivative $\frac{dP(r)}{dr}$ to determine the value of r such that $P(r)$ has its maximum value (you can leave your answer in terms of the Bohr radius); in other words, the most probable location for the electron.

80: **Integrating Radial Probability Distribution.** The radial probability distribution function $P(r)$ of the ground state electron in the hydrogen atom is given in the previous problem. The normalization condition is $\int_0^\infty P(r)dr = \int_0^\infty Cr^2 e^{-2r/a_0} dr = 1$, which just indicates that the electron must be found ***somewhere*** in the region $0 \le r \le \infty$. Determine the value for the constant C; you can leave your answer in terms of the Bohr radius and constants. Evaluate the integral by hand using your integral table; I suspect your calculator may not be able to do this integral. (Hint: as $x \to \infty$, e^{-x} goes to zero faster than x^2 blows up.)

81: **Visualizing Radial Probability Distributions.** The following web exercises will help you to plot the radial probability distributions for the electron in a hydrogen atom. They are from the Davidson College physics department.

a) Go to www.phy.Davidson.edu/StuHome/cabell_f/Radial.html

This program graphs the radial probability density $P(r) \propto r^2|\psi|^2$ vs. r. The graph that shows up first should be for the $n = 1, \ell = 0$ state. Verify that this is the same shape as Figure 36-10 in your book.

b) Click on the button Choose Own States. The boxes should be filled in with $n = 1$ and $\ell = 0$. Enter 1 ℓ in the box, and click on the Plot button. You should notice that the box you just filled in turns red, which indicates an error. What is the problem with what you were asked to do?

c) Sketch a plot for the 1s state (the one you just saw). How many nodes do you see (not counting $r = 0$ and r = infinity)? How many antinodes? Where is the antinode (note that the horizontal axis is in terms of r/a_0)? Remember the antinode represents the place of maximum likelihood of finding the particle. Compare this to the result predicted by Bohr, who said that the electron makes a circular orbit of fixed radius $r_n = n^2 a_0$.

d) Sketch the plots for the 2s and 2p states. How many nodes does the 2s state have? Where is the antinode for the 2p state? How does this compare to the Bohr radius for this state? Repeat for the 3s, 3p and 3d states. Take specific note of the location of the antinode for the 3d state. How does this location compare to the Bohr radius for this state?

e) Hopefully, you've noticed a trend between the location of the maximum probability for one of the states for a particular n and the Bohr radius for that state. Use this to predict the location of the antinode for the 4f state. Verify your prediction using the program. (It is interesting to note that despite all the ad hoc assumptions that went into Bohr's semi-classical model of the hydrogen atom, he got some of the same features that come out of the quantum theory of the hydrogen atom as obtained by Schroedinger; definitely not all of them, but some).

f) Consider the 2s state again. You should have seen that there was one node. This should mean that the probability of finding the electron in a spherical shell centered about this node should be zero. Go to the related web site: www.phy.Davidson.edu/StuHome/cabell_f/Density.html

Obtain a plot for the 2s state. Do you see a spherical shell where there is no probability of finding the electron?

82: **21-cm Hydrogen Line.** The proton, like the electron, has spin, which means the proton is the source of a magnetic field which the electron feels. In the hydrogen atom in its ground state ($n = 1, \ell = 0$), there are actually two energy levels, depending on whether the electron and proton spins are parallel or anti-parallel. If the electron of an atom has a spin flip from the state of higher energy to that of lower energy, a photon of wavelength 21 cm (in the microwave band) is emitted. Radio astronomers observe this 21 cm radiation coming from deep space. What is the effective magnetic field (created by the proton) experienced by the electron emitting this radiation?

83: **Proton Spin-flip?** In a magnetic resonance imaging scan, the magnetic field at a particular point has a magnitude 0.50 T. The frequency of an EM field oscillation is 33 MHz. Determine if proton spins at this particular location flip in response to this field.